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Chapter 1 : Deformation Quantization-A Brief Survey | Gizem Karaali - calendrierdelascience.com

Deformation quantization Intuitively, a deformation of a mathematical object is a family of the same kind of objects that depend on some parameter(s). Here, it provides rules for how to deform the "classical" commutative algebra of observables to a quantum non-commutative algebra of observables.

Introduction Quantization is, most broadly, the process of forming a quantum me- chanical system starting from a classical mechanical one. See Be for an early attempt to obtain a general definition of quantization. AbM also provides an introductory account of the subject. There are various methods of quantization; see BW for a general introduction to the geometry of quantization, and a specific geometric method geometric quantization. In this survey we will be interested in deformation quantization. In- tuitively a deformation of a mathematical object is a family of the same kind of objects depending on some parameter s. The deforma- tion of algebras is central to our problem, and in particular we are con- cerned with the deformations of function algebras. In the first section of this paper we will give a short overview of the main ideas in deformation quantization. In the second section we will go into more historical details about the directions in which the area developed in the last 20 years and in the last section we will try to give a very sketchy summary of recent results by Kontsevich. An Overview The basic setup in deformation theory is as follows: We start with an algebraic structure, e. Fl The question also has a purely mathematical interest when considered in the most abstract sense, but here we will be mainly interested in the more specific questions which arise within the subject of quantization. The traditional quantum formalism on the other hand interprets observables as certain operators on a Hilbert space and these do not commute! Instead of forcing quantization to involve such a radical change in the nature of the observables, the authors of the influential papers BFFLS1,BFFLS2 suggested that it be understood as a deformation of the structure of the algebra of classical observables. Some Definitions and Preliminaries: A formal deformation of the Poisson bracket is a skew- symmetric map [,]: For variations of these definitions see Br. In classical mechanics the phase space M is the cotangent bundle of the configuration space which is a smooth manifold. Can we find a formal deformation of the function algebra of an arbitrary Poisson manifold? In such a case we would have: See BW , Ka or W for more details. Other Directions in Deformation Quantization. The question of existence of star-products has been extensively stud- ied. We will see in the next section a short account of these develop- ments. For a more detailed account see Br. For a survey of this research see Bh. Historical Development The results about deformation quantization came in gradually, as existence proofs in increasing levels of generality. Even more general results could be obtained by coho- mological methods G , GeS. Here we digress slightly to give some definitions for the terms used above: Let A be an associative algebra over some commutative ring K and for simplicity assume it is a module over itself with the adjoint action i. Let Z p A, A be the space of all p-cocyles and B p A, A the space of those p-cocycles that are coboundaries of a p-1 -cochain. The p-cochains here are skew-symmetric, i. Again this is a complex, i. Thus the cocycles space Zcp , the coboundaries space Bcp , and the quotient space Hcp A, A or Hcp A in short can be defined, analogously.. We can now return to our short historical account: With the above mentioned tools, first, in mid s, the existence of star-products for symplectic manifolds whose third cohomology group is trivial was proved, but this restriction turned out to be merely tech- nical. A further generalization was achieved with Fe where Fedosov proved that the results about the canonical star-product on an arbitrary symplectic manifold can be used to prove that all regular Poisson manifolds can be quantized. However the question at the end of Section 2. In physics we sometimes require manifolds which have a degenerate Pois- son bracket and so are not symplectic. The broadest framework for classical mechanics thus involves general Poisson manifolds. Therefore all the results mentioned above provided only a partial answer to the problem of quantization. The Formality Conjecture is proved in Ko3 thus answering our question in 2. In the last part of this paper we will try to summarize these results in a very sketchy manner, I am afraid! The cohomological arguments were introduced before, as we have seen above. In his work about quantization of Poisson manifolds, Kont- sevich also made use of cohomology. However, his approach involved further concepts that we will be introducing in 4. Some More Definitions and Preliminaries: In see Ko1 and Ko2 ,

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Kontsevich proposed the fol- lowing Conjecture 1. In other words he proved the following Theorem 2. The Formality Conjecture for a manifold M implies de- formation quantization of any Poisson structure on M. Then there is a natural isomorphism between equivalence classes of defor- mations of the null Poisson structure on M and equivalence classes of smooth deformations of the associative algebra A. In particular any Poisson bracket on M comes from a canonically defined modulo equiv- alence star product. Moreover, our prob- lem is solved: For the theorem implies the conjecture which implies the existence of deformation quan- tization. Final Results and Implications. In this part we will be mainly following St and Ko4. A later result shows that in addition to the existence of a canonical way of quantization, we can define a universal infinite-dimensional manifold parametrizing quantizations. Now it can be seen that Kontsevich has proved a more general result than the existence of deformation quantization of any Poisson manifold. He has proved that in a suitably defined homotopy category of DGLAs two objects are equivalent. The first object is the Hochschild complex of the algebra of functions on the manifold M, and the second is a Z-graded Lie superalgebra of multivector fields on M. Although he has some doubts as to the naturality of the method for quantum mechanics Kontsevich seems to believe that the result shows there is some intrinsic relation with string theory. See remarks in 1. Thus even though the original question that arose at the end of 2. For instance a natural conjecture that could follow the above results could be: An- other relation to be considered could be that of the result with some 2-cohomology on the manifold. In a different direction we might consider a remark of Kontsevich: Ko4 Another question that remains involves the infinite dimensional case: This case involves new problems and perhaps may shed light on a better mathematical understanding of quantum field theory. NY , , , , Be Berezin, F. B32, , G Gerstenhaber, M. C , , Kluwer Acad.

Chapter 2 : Fedosov : A simple geometrical construction of deformation quantization

Idea. Deformation quantization is one formalization of the general idea of quantization of a classical mechanical system/classical field theory to a quantum mechanical system/quantum field theory.

Chapter 3 : C* algebraic deformation quantization in nLab

Deformation quantization of a Poisson algebra. Given a Poisson algebra (A, {â‹…, â‹…}), a deformation quantization is an associative unital product â˜… on the algebra of formal power series in Ä§, A[[Ä§]], subject to the following two axioms.

Chapter 4 : Wignerâ€“Weyl transform - Wikipedia

NOTES ON DEFORMATION QUANTIZATION 3 rewrite the associativity condition of ~ in terms of a DGLA structure on the Hochschild complex (C (A;A);b).Let Ëšbe a Hochschild p-cochain and 'be a Hochschild q-cochain.

Chapter 5 : Kontsevich quantization formula - Wikipedia

Deformation quantization of Poisson manifolds Maxim Kontsevich Foreword Here is the ï¬•nal version of the e-print Deformation quantization of Poisson manifolds I [33].

Chapter 6 : Topics: Deformation Quantization

DEFORMATION QUANTIZATION - A BRIEF SURVEY GIZEM KARAALI 1. Introduction Quantization is, most broadly, the process of forming a quantum me-chanical system starting from a classical mechanical one.

Chapter 7 : Deformation quantization: a survey - IOPscience

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*Deformation Quantization: In General > s.a. geometric quantization; quantum group; schrÃ¶dinger equation [generalizations]; Star Product. * Idea: An approach to quantization in which the classical algebra of observables for a physical systems is replaced by a deformed algebra, with multiplication replaced by a (non-commutative but associative) star product; The best known example is the Moyal.*

Chapter 8 : deformation quantization in nLab

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Chapter 9 : Newest 'deformation-quantization' Questions - Physics Stack Exchange

the sense of deformation quantization). Informally, it means that the set of equivalence classes of associative algebrasclose to algebrasof functions on manifolds is in one-to-onecorrespondencewith the set of equivalence.