Chapter 1 : MA Differential Equations

In total the reader will find a valuable guide to over selected references that use differential equations, stability analysis and chaotic dynamics. Graduate students in economics with a special interest in economic theory, economic researchers and applied mathematicians will all benefit from this volume.

Index PREFACE As a formal model of an economy acquires a mathematical life of its own, it becomes the object of an inexorable process in which rigor, generality and simplicity are relentlessly pursued. As the title of the book indicates, the mathematical methods presented are ordinary differential equations, stability techniques and chaotic dynamics. The applications selected to illustrate these methods are numerous and include microeconomic dynamics, investment theory, macroeconomic policies, capital theory, business cycles, financial economics and many others. The use of ordinary differential equations in economics, dates back, at least to L. During the past forty years, ordinary differential equations have been employed extensively by economic researchers and such a use has created the need for an exposition of the fundamental notions and properties of these equations. This is done in chapters 1 and 2; an emphasis on existence, continuation of solutions, uniqueness, successive approximations and dependence on initial data and parameters is given in chapter 1. Chapter 2 discusses linear differential equations with a balanced approach between their properties and solutions. We note that although the classical approach to differential equations concentrates on methods and techniques for finding explicit solutions, the xii Preface modern approach endeavors to obtain information about the whole class of solutions and their properties. Thus, the emphasis of chapter 1 is on properties while chapter 2 balances abstraction with concreteness and illustrates some problem solving techniques. The notion of stability in economics, introduced by Walras and Cournot and later studied by Marshall, Hicks, Samuelson and others, was not completely formulated until the late s in the papers of Arrow and his collaborators. The last thirty years have seen numerous papers on stability analysis applied to economic models. However, stability methods continue to remain under-utilized by economists partly because our profession has not been taught these methods in any detail and partly because the critical role played by stability analysis has not been fully appreciated outside theoretical circles. Chapter 3 gives numerous definitions and examples of stability notions, and discusses the stability properties of linear dynamic systems. It also gives a comprehensive presentation of two dimensional systems and their phase diagrams. Chapter 4 continues on the topic of stability at a more advanced level. Liapunov theory for local stability and global asymptotic stability receive the bulk of our attention. In other words, chapter 3 illustrates the significance of linear systems, the linearization of nonlinear systems, the counting and examination of roots of characteristic equations while chapter 4 is more topological in nature. Furthermore, the recent results of several mathematicians, such as Hartman and Olech , Markus and Yamabe and Olech on global asymptotic stability receive special attention in a unique way which has not previously been done by mathematical economics textbooks. The above is not all we have to say on stability methods. An additional important contribution is chapter 5. This chapter surveys several important methods of stability analysis of optimal control problems. While chapters 3 and 4 are a collection of stability results contributed by mathematicians that were essentially completed prior to , chapter 5 surveys the stability contributions of mathematical economists, all of which took place since the mid s. This chapter unites the economic analyst and the mathematician at the forefront of stability research and is used as a foundation for later chapters. Through the contents of this book we hope to show that the stability analysis of an economic model is an integral part of economic research. The stability properties of an economic model must be investigated and become understood before such a model is used to supply insights into the workings of the actual economic system. Although conditions of the Preface xiii existence and uniqueness of equilibrium are investigated almost automatically in economic model building, it is not always true that the same is done for stability. The many stability methods discussed in this book and the numerous illustrations are intended to persuade the economic theorist to become more comprehensive in his or her analysis of economic models. To encourage this methodological approach, stability is viewed as a property of the solution of the differential equation and chapter 3 makes explicit the connection between the property of dependence on initial data and the concept of stability. Having argued that differential equations abound in economics and that stability methods form a limited subset of the former, what can we say about chaotic dynamics? Methods related to chaos and nonlinearity are very new to economics. As chapter 10 documents, the number of economic papers in this area, although growing very rapidly, remains limited, with most of these papers having been written in the s. Chapter 10 attempts to give a mathematically precise version of recent tests on an observed time series for the presence of low dimensional deterministic chaos. A simple methodological connection between stability and chaos explains why chaos is introduced in this book. Chaos theory removes the emphasis from stability by stressing instabilities. The exposition of differential equations, stability and chaos as a collection of important mathematical methods has not been made at the expense of illustrations. Fully one half of this book is devoted to applications. More specifically, chapter 6 is written to make two points. First, an infinite horizon two sector economy in which one sector is decreasing returns and the other is increasing returns may fail to achieve Pareto optimum, under decentralized institutions. This may occur even when the increasing returns sector is regulated in a first best fashion, efficient markets prevail, rational expectations obtain, and the necessity of the transversality condition at infinity for identical infinitely lived agents eliminate Hahn type problems. Second, there is a tendency for the increasing returns sector to overexpand although this is not always the case. The chapter also provides an analytically tractable framework where the impact of different modes of regulation upon economic development paths may be studied. Multiple optimal paths are also investigated. Chapter 7 addresses stability issues in investment theory with primary emphasis on cost of adjustment models. Instead of just citing results, this chapter proposes a novel way of studying stability as a methodological consequence of a modified Correspondence Principle. The famous original Correspondence Principle of Paul Samuelson is discussed and a xiv Preface modification is proposed and utilized to obtain stability results in investment models. Chapter 8 extends some of the recent macrodynamic models in two ways. First, it specifies a more complete corporate sector that such models usually contain and, second the relationships describing the private sector are derived from explicit optimizing procedures by households and firms. The equilibrium structure and dynamics of this model are studied in detail and the stability of a simplified case is analyzed. Chapter 9 applies the results of chapters 1 through 5 to capital theory. A capital theory generates a capital-price differential equation by using the Pontryagin maximum principle to write the necessary conditions for an optimal solution. This process generates a system of differential equations that is called a modified Hamiltonian dynamical system. This chapter analyzes the stability properties of such modified Hamiltonian dynamical systems. All chapters conclude with two sections on miscellaneous applications and exercises and further remarks and references. In total the reader will find a valuable guide to over selected references that use differential equations, stability analysis and chaotic dynamics. We are quite certain that this is the first economics monograph of its kind offering the economic theorist the opportunity to acquire new and important analytical tools. There are currently no books available covering all the methods presented here; nor are there any books with such a comprehensive coverage of applications. The primary audience of this book will include PhD students in economics with a special interest in economic theory. Furthermore, economic researchers should benefit from this book by developing expertise in the methods studied. Finally, applied mathematicians will find fresh mathematical ideas in chapter 5 relating to the stability of optimal control and can broaden the domain of their stability and chaos examples from chapters 6 through An attempt has been made to keep the mathematical background to a minimum. Many parts of this book can be understood by someone with a good background in analysis. The Appendix cites numerous definitions and theorems to help readers with insufficient mathematical background and gives references for further study. They gave us the original, creative and valuable contributions which form the substance of this book. Becker Indiana University , E. Burmeister University of Virginia , F. Chang Indiana University and S. Turnovsky University of Washington wrote detailed comments that helped us improve our exposition. Numerous individuals helped through insightful comments, suggestions, encouragement and interest in our work and among them we mention: Barron Loyola University of Chicago , J. Benhabib New York University , J. Constantinides University of Chicago , D. Dechert University of Iowa , M. Hadjimichalakis University of Washington , K. Judd University of Chicago , M. Magill University of Southern California , D. Meyer Loyola University of Chicago , J. Scheinkman University of Chicago , S. Stefani University of Brescia, Italy , A. Takayama University of Southern Illinois and anonymous referees. Several research assistants helped with proofreading, editing and bibliographical work. Pamela Kellman made numerous editorial suggestions that improved the presentation of our ideas. Carmela Perno has shown outstanding patience and extraordinary skills in typing various versions during a period of five years. North-Holland , have been a pleasure to work with. We are thankful to the Academic Press and the editors of the Journal of Economic Theory, to Springer-Verlag, to the North-Holland Publishing xvi Acknowledgements Company and to the editors of the International Economic Review for giving us permission to use copyrighted papers authored and coauthored by W. Parts of the book have been used by numerous students at the University of Wisconsin, Stanford University, Indiana University and Loyola University of Chicago and we are grateful to all students who have read and commented on the manuscript. Brock dedicates his portion of this book to his wife Joan "who makes his work possible and his life fun" and A. Malliaris dedicates his portion to his wife Mary Elaine and their children Maryanthe and Steven for "their love, patience and joy". Hirsch and Smale , p. Introduction The study of ordinary differential equations may be pursued from at least two broadly distinct approaches. On the one hand, one may endeavor to learn a large number of methods and techniques by which certain elementary equations can be solved explicitly. Alternatively, one may concentrate on obtaining information about the whole class of solutions and their properties, putting aside all endeavors to master skillfully a myriad of methods yielding closed form solutions. In this chapter we select the latter approach to study the basic properties of ordinary differential equations and their solutions without expositing methods of solution. More specifically, we discuss in some detail the following topics: Actually, the important property of stability is introduced in this chapter and then left to be treated in some detail in the following chapters. There are at least three reasons justifying our approach in this chapter, with its emphasis on properties of solutions rather than on methods yielding explicit solutions. First, the notion of stability and its applications both seek to obtain information about a property of the whole class of solutions. Second, almost all economic applications of stability exposited in this book or found in the general literature concentrate on the property of stability which is not dependent upon a specific and explicit solution of a differential equation. Thus most applications strive for generality and so does our analysis. Finally, our approach illustrates the modern inclination and interest of mathematicians with preference for abstraction and generality rather than amplification and specification.

Chapter 2 : Differential Equations in Economics - calendrierdelascience.com

Differential Equations, Stability and Chaos in Dynamic Economics has 1 rating and 0 reviews. This is the first economics work of its kind offering the ec.

Applications of differential equations are now used in modeling motion and change in all areas of First, it provides a comprehensive introduction to most important concepts and theorems in differential Chapter 1 Differential Equations in Economics Applications of differential equations are now used in modeling motion and change in all areas of science. The theory of differential equations has become an essential tool of economic analysis particularly since computer has become commonly available. It would be difficult to comprehend the contemporary literature of economics if one does not understand basic concepts such as bifurcations and chaos and results of modem theory of differential equations. A differential equation expresses the rate of change of the current state as a function of the current state. Consider state x of the GDP of the economy. The growth rate of the GDP is xlx. If the growth rate g is given at any time t, the GDP at t is given by solving the differential equation. The solution tells that the GDP decays increases exponentially in time when g is negative positive. We can explicitly solve the above differential function when g is a constant. This means that the growth rate may take on a complicated form g x, t. There are various established methods of solving different types of differential equations. This book introduces concepts, theorems, and methods in differential equation theory which are widely used in contemporary economic analysis and provides many simple as well as comprehensive applications to different fields in economics. This book is mainly concerned with ordinary dzflerential equations. Ordinary differential equations are differential equations whose solutions are functions of one independent variable, which we usually denote by t. The variable t often stands for time, and solution we are looking for, x t , usually stands for some economic quantity that changes with time. Therefore we consider x t as a dependent variable. Ordinary differential equations are classified as autonomous and nonautonomous. If the equation specially involves t, we call the equation nonautonomous or timedependent. In this book, we often omit "ordinary", "autonomous", or "nonautonomous" in expression. If an equation involves derivatives up to and includes the ith derivative, it is called an ith order differential equation. The first derivative x is the only one that can appear in a first order differential equation, but it may enter in various powers: The highest power attained by the derivative in the equation is referred to as the degree of the differential equation. First, it provides a comprehensive introduction to most important concepts and theorems in differential equations theory in a way that can be understood by anyone who has basic knowledge of calculus and linear algebra. In addition to traditional applications of the theory to economic dynamics, this book also contains many recent developments in different fields of economics. The book is mainly concerned with how differential equations can be applied to solve and provide insights into economic dynamics. We emphasize "skills" for application. When applying the theory to economics, we outline the economic problem to be solved and then derive differential equation s for this problem. These equations are then analyzed andlor simulated. Different from most standard textbooks on mathematical economics, we use computer simulation to demonstrate motion of economic systems. A large fraction of examples in this book are simulated with Mathematica. Today, more and more researchers and educators are using computer tools such as Mathematica to solve - once seemingly The nth derivative of x t ,denoted by d n t is, the derivative of x "-" t. This book provides not only a comprehensive introduction to applications of linear and linearized differential equation theory to economic analysis, but also studies nonlinear dynarnical systems which have been widely applied to economic analysis only in recent years. Linearity means that the rule that determines what a piece of a system is going to do next is not influenced by what it is doing now. The mathematics of linear systems exhibits a simple geometry. The simplicity allows us to capture the essence of the problem. Nonlinear dynamics is concerned with the study of systems whose time evolution equations are nonlinear. If a parameter that describes a linear system, is changed, the qualitative nature of the behavior remains the same.

But for nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the quantitative and qualitative behavior of the system. Nonlinear dynamical theory reveals how such interactions can bring about qualitatively new structures and how the whole is related to and different from its individual components. The study of nonlinear dynamical theory has been enhanced with developments in computer technology. A modern computer can explore a far wider class of phenomena than it could have been imagined even a few decades ago. The essential ideas about complexity have found wide applications among a wide range of scientific disciplines, including physics, biology, ecology, psychology, cognitive science, economics and sociology. Many complex systems constructed in those scientific areas have been found to share many common properties. The great variety of applied fields manifests a possibly unifying methodological factor in the sciences. Nonlinear theory is bringing scientists closer as they explore common structures of different systems. It offers scientists a new tool for exploring and modeling the complexity of nature and society. The new techniques and concepts provide powerful methods for modeling and simulating trajectories of sudden and irreversible change in social and natural systems. Modem nonlinear theory begins with Poincard who revolutionized the study of nonlinear differential equations by introducing the qualitative techniques of geometry and topology rather than strict Differential Equations in Economics 5 analytic methods to discuss the global properties of solutions of these systems. He considered it more important to have a global understanding of the gross behavior of all solutions of the system than the local behavior of particular, analytically precise solutions. The study of the dynamic systems was furthered in the Soviet Union, by mathematicians such as Liapunov, Pontryagin, Andronov, and others. Around , many scientists around the world were suddenly aware that there is a new kind of motion - now called chaos - in dynamic systems. The new motion is erratic, but not simply "quasiperiodic" with a large number of periods. What is surprising is that chaos can occur even in a very simple system. Scientists were interested in complicated motion of dynamic systems. But only with the advent of computers, with screens capable of displaying graphics, have scientists been able to see that many nonlinear dynamic systems have chaotic solutions. As demonstrated in this book, nonlinear dynamical theory has found wide applications in different fields of economics. The range of applications includes many topics, such as catastrophes, bifurcations, trade cycles, economic chaos, urban pattern formation, sexual division of labor and economic development, economic growth, values and family structure, the role of stochastic noise upon socio-economic structures, fast and slow socio-economic processes, and relationship between microscopic and macroscopic structures. All these topics cannot be effectively examined by traditional analytical methods which are concerned with linearity, stability and static equilibria. For instance, the traditional view of the relations between laws and consequences - between cause and effect - holds that simple rules imply simple behavior, therefore complicated behavior must arise from - -- In the solar system, the motion traveled around the earth in month, the earth around the sun in about a year, and Jupiter around the sun in about Such systems with multiple incommensurable periods are known as quasiperiodic. For recent applications of nonlinear theory to economics, see Rosser , Zhang , , Lorenz , Puu , and Shone This vision had been held by professional economists for a long time. But it has been recently challenged due to the development of nonlinear theory. Nonlinear theory shows how complicated behavior may arise from simple rules. This is a simple dynamical system. The model with the specified parameter values does not exhibit any regular or periodic behavioral pattern. Chaos persists for as long as time passes. Another example is the Lorenz equations. The laws that govern the motion of air molecules and of other physical quantities are well known. The topic of differential equations is some years old, but nobody would have thought it possible that differential equations could behave as chaotically as Edward N. Lorenz found in his experiments. Around , Lorenz constructed models for numerical weather forecasting. He showed that deterministic natural laws do not exclude the possibility of chaos. In other words, determinism and predictability are not equivalent. In fact, recent chaos theory shows that deterministic chaos can be identified in much simpler systems than the Lorenz model. There are two sheets in which trajectories spiral outwards. When the distance from the center of such a spiral becomes larger than some particular threshold, the motion is ejected from the spiral and is attracted by the other spiral, where it again begins to spiral out, and the process is repeated. The motion is not regular. The number of turns that a trajectory spends in one spiral before it jumps to the other is not specified. It may wind around one spiral twice, and then three times around the other, then ten times around the first and so on. Nonlinear dynamical systems are sufficient to determine the behavior in the sense that solutions of the equations do exist. But it is often A very thorough treatment of the Lorenz equations is given by Sparrow 1 Nonlinear economics based on nonlinear dynamical theory attempts to provide a new vision of economic dynamics: There is a tendency to replace simplicity with complexity and specialism with generality in economic research. The concepts such as totality, nonlinearity, self-organization, structural changes, order and chaos have found broad and new meanings by the development of this new science. According to this new science, economic dynamics are considered to resemble a turbulent movement of liquid in which varied and relatively stable forms of current and whirlpools constantly change one another. These changes consist of dynamic processes of self-organization along with the spontaneous formation of increasingly subtle and complicated structures. The accidental nature and the presence of structural changes like catastrophes and bifurcations, which are characteristic of nonlinear systems and whose further trajectory is determined by chance, make dynamics irreversible. Traditional economists were mainly concerned with regular motion of the economic systems. Even when they are concerned with economic dynamics, students are still mostly limited to their investigations of differential or difference equations to regular solutions which include steady states and periodic solutions. In particular, economists were mainly interested in existence of a unique stable equilibrium. Students trained in traditional economics tend to imagine that the economic reality is uniquely determined and will remain invariant over time under "ideal conditions" of preferences, technology, and institutions. Nevertheless, common experiences reveal more complicated pictures of economic reality. Economic structures change even within a single generation. Economic systems collapse or suddenly grow without any seemingly apparent signs of structural changes. It is of potential interest to professionals and graduate students in economics and applied mathematics, as well as researchers in social sciences with an interest in applications of differential equations to economic systems. The book is basically divided into three parts - Part I concerns with one-dimensional differential equations; Part I1 concentrates on planar differential equations; Part I11 studies higher dimensional dynamical systems.

DOWNLOAD PDF DIFFERENTIAL EQUATIONS, STABILITY, AND CHAOS IN DYNAMIC ECONOMICS

Chapter 3 : Differential Equations, Stability and Chaos in Dynamic Economics by W.A. Brock

Part 1 approximately covers a one-semester course in ordinary differential equations with emphasis on stability theory and is intended to provide the prerequisites for the second part. Part 2 is a collection of dynamic economic models analysing several of the most puzzling and fascinating issues in today's mathematical economics.

Handbook of Computational Economics Vol. Stability Theorems with Economic Applications. Econometrica, 45 2 , Econometrica, 44 3 , Stability of Time-Delay Systems. Extraction of non-renewable resources: Dynamic optimization in natural resources management. Journal of Environmental Management and Tourism I 2 2 , Cyclical and constant strategies in renewable resources extraction. Mathematics for Economics, Second Edition. Mathematics and Mathematica for Economists. Oxford Blackwell Publishers, Massachusetts. Handbook of computational economics: Agent-based computational economics Vol. Inners and Stability of Dynamic Systems. Programming Languages in Economics. Computational Economics, 14, The Degree of Damping in Business Cycles. Econometrica, 8 1 , Applied Computational Economics and Finance. Weak stability for matrices. Linear and Multilinear Algebra, 7, â€” Computational Economics, 31, Linear Algebra and its Applications 3rd edition. Economic and financial analysis with mathematica. Differential Equations, Bifurcations, and Chaos in Economics.

Chapter 4 : Stability analysis in economic dynamics: A computational approach - Munich Personal RePEc A

Book Reviews Differential Equations, Stability and Chaos in Dynamic Economics. By WILLIAM A. BROCK and A. G. MALLIARIS. Amsterdam: North-Holland, Pp. xvi+

Overview[edit] The concept of a dynamical system has its origins in Newtonian mechanics. There, as in other natural sciences and engineering disciplines, the evolution rule of dynamical systems is an implicit relation that gives the state of the system for only a short time into the future. The relation is either a differential equation , difference equation or other time scale. To determine the state for all future times requires iterating the relation many timesâ€"each advancing time a small step. The iteration procedure is referred to as solving the system or integrating the system. If the system can be solved, given an initial point it is possible to determine all its future positions, a collection of points known as a trajectory or orbit. Before the advent of computers , finding an orbit required sophisticated mathematical techniques and could be accomplished only for a small class of dynamical systems. Numerical methods implemented on electronic computing machines have simplified the task of determining the orbits of a dynamical system. For simple dynamical systems, knowing the trajectory is often sufficient, but most dynamical systems are too complicated to be understood in terms of individual trajectories. The difficulties arise because: The systems studied may only be known approximatelyâ€"the parameters of the system may not be known precisely or terms may be missing from the equations. The approximations used bring into question the validity or relevance of numerical solutions. To address these questions several notions of stability have been introduced in the study of dynamical systems, such as Lyapunov stability or structural stability. The stability of the dynamical system implies that there is a class of models or initial conditions for which the trajectories would be equivalent. The operation for comparing orbits to establish their equivalence changes with the different notions of stability. The type of trajectory may be more important than one particular trajectory. Some trajectories may be periodic, whereas others may wander through many different states of the system. Applications often require enumerating these classes or maintaining the system within one class. Classifying all possible trajectories has led to the qualitative study of dynamical systems, that is, properties that do not change under coordinate changes. Linear dynamical systems and systems that have two numbers describing a state are examples of dynamical systems where the possible classes of orbits are understood. The behavior of trajectories as a function of a parameter may be what is needed for an application. As a parameter is varied, the dynamical systems may have bifurcation points where the qualitative behavior of the dynamical system changes. For example, it may go from having only periodic motions to apparently erratic behavior, as in the transition to turbulence of a fluid. The trajectories of the system may appear erratic, as if random. In these cases it may be necessary to compute averages using one very long trajectory or many different trajectories. The averages are well defined for ergodic systems and a more detailed understanding has been worked out for hyperbolic systems. Understanding the probabilistic aspects of dynamical systems has helped establish the foundations of statistical mechanics and of chaos. In them, he successfully applied the results of their research to the problem of the motion of three bodies and studied in detail the behavior of solutions frequency, stability, asymptotic, and so on. Aleksandr Lyapunov developed many important approximation methods. His methods, which he developed in , make it possible to define the stability of sets of ordinary differential equations. He created the modern theory of the stability of a dynamic system. Combining insights from physics on the ergodic hypothesis with measure theory , this theorem solved, at least in principle, a fundamental problem of statistical mechanics. The ergodic theorem has also had repercussions for dynamics. Stephen Smale made significant advances as well. His first contribution is the Smale horseshoe that jumpstarted significant research in dynamical systems. He also outlined a research program carried out by many others. The notion of smoothness changes with applications and the type of manifold. When T is taken to be the reals, the dynamical system is called a flow ; and if T is restricted to the non-negative reals, then the dynamical system is a semi-flow. When

T is taken to be the integers, it is a cascade or a map; and the restriction to the non-negative integers is a semi-cascade.

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Chapter 5 : Attractors, Bifurcations, & Chaos: Nonlinear Phenomena in Economics - TÃ¶nu Puu - Google E

Differential Equations, Stability and Chaos in Dynamic Economics (Advanced Textbooks in Economics.

The output of op amp 0 will correspond to the x variable, the output of 1 corresponds to the first derivative of x and the output of 2 corresponds to the second derivative. Spontaneous order[edit] Under the right conditions, chaos spontaneously evolves into a lockstep pattern. In the Kuramoto model , four conditions suffice to produce synchronization in a chaotic system. Natural forms ferns, clouds, mountains, etc. In the s, while studying the three-body problem , he found that there can be orbits that are nonperiodic, and yet not forever increasing nor approaching a fixed point. Chaos theory began in the field of ergodic theory. Despite initial insights in the first half of the twentieth century, chaos theory became formalized as such only after mid-century, when it first became evident to some scientists that linear theory , the prevailing system theory at that time, simply could not explain the observed behavior of certain experiments like that of the logistic map. What had been attributed to measure imprecision and simple " noise " was considered by chaos theorists as a full component of the studied systems. The main catalyst for the development of chaos theory was the electronic computer. Much of the mathematics of chaos theory involves the repeated iteration of simple mathematical formulas, which would be impractical to do by hand. Electronic computers made these repeated calculations practical, while figures and images made it possible to visualize these systems. Yet his advisor did not agree with his conclusions at the time, and did not allow him to report his findings until Studies of the critical point beyond which a system creates turbulence were important for chaos theory, analyzed for example by the Soviet physicist Lev Landau , who developed the Landau-Hopf theory of turbulence. David Ruelle and Floris Takens later predicted, against Landau, that fluid turbulence could develop through a strange attractor , a main concept of chaos theory. Edward Lorenz was an early pioneer of the theory. His interest in chaos came about accidentally through his work on weather prediction in He wanted to see a sequence of data again, and to save time he started the simulation in the middle of its course. He did this by entering a printout of the data that corresponded to conditions in the middle of the original simulation. To his surprise, the weather the machine began to predict was completely different from the previous calculation. Lorenz tracked this down to the computer printout. The computer worked with 6-digit precision, but the printout rounded variables off to a 3-digit number, so a value like 0. This difference is tiny, and the consensus at the time would have been that it should have no practical effect. However, Lorenz discovered that small changes in initial conditions produced large changes in long-term outcome. In , Benoit Mandelbrot found recurring patterns at every scale in data on cotton prices. In , he published " How long is the coast of Britain? In , Mandelbrot published The Fractal Geometry of Nature , which became a classic of chaos theory. Yorke coiner of the term "chaos" as used in mathematics , Robert Shaw , and the meteorologist Edward Lorenz. In , Albert J. Feigenbaum for their inspiring achievements. There, Bernardo Huberman presented a mathematical model of the eye tracking disorder among schizophrenics. In , Per Bak , Chao Tang and Kurt Wiesenfeld published a paper in Physical Review Letters [59] describing for the first time self-organized criticality SOC , considered one of the mechanisms by which complexity arises in nature. Alongside largely lab-based approaches such as the Bakâ€"Tangâ€"Wiesenfeld sandpile , many other investigations have focused on large-scale natural or social systems that are known or suspected to display scale-invariant behavior. Although these approaches were not always welcomed at least initially by specialists in the subjects examined, SOC has nevertheless become established as a strong candidate for explaining a number of natural phenomena, including earthquakes , which, long before SOC was discovered, were known as a source of scale-invariant behavior such as the Gutenbergâ€"Richter law describing the statistical distribution of earthquake sizes, and the Omori law [60] describing the frequency of aftershocks , solar flares , fluctuations in economic systems such as financial markets references to SOC are common in econophysics , landscape formation , forest fires , landslides , epidemics , and biological evolution where SOC has been invoked, for example, as the dynamical mechanism behind the theory of " punctuated equilibria " put forward by Niles Eldredge and Stephen Jay Gould. Given the implications of a scale-free distribution of event sizes, some researchers have suggested that another phenomenon that should be considered an example of SOC is the occurrence of wars. In the same year, James Gleick published Chaos: Making a New Science , which became a best-seller and introduced the general principles of chaos theory as well as its history to the broad public, though his history under-emphasized important Soviet contributions.

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Chapter 7 : Chaos theory - Wikipedia

2 Differentia/ Equations, Bifurcations, and Chaos in Economics many other conditions. This means that the growth rate may take on a complicated form g(x, t). The economic growth is described by.

Chapter 8 : Dynamical system - Wikipedia

Differential equations, stability and chaos in dynamic economics Routh-Hurwitz criterion. The advantage of the Liapunov method becomes evident when, instead of considering a linear system as (), we apply the Liapunov method of lemma to the equation Recall that () is the same as () of the previous chapter.