Chapter 1 : Oprea, Differential Geometry and Its Applications, 2nd Edition | Pearson

This book studies the differential geometry of surfaces and aims to help students make the transition from the standard university curriculum to a type of mathematics that is a unified whole, by mixing geometry, calculus, linear algebra, differential equations, complex variables, the calculus of variations, and notions from the sciences.

Furthermore, how can we ameliorate the quantum leap from introductory calculus and linear algebra to more abstract methods in both pure and applied mathematics? There is a subject which can take students of mathematics to the next level of development and this subject is, at once, intuitive, calculable, useful, interdisciplinary and, most importantly, interesting. Differential Geometry provides the perfect transition course to higher mathematics and its applications. It is a subject which allows students to see mathematics for what it is - not the compartmentalized courses of a standard university curriculum, but a unified whole mixing together geometry, calculus, linear algebra, differential equations, complex variables, the calculus of variations and various notions from the sciences. Moreover, Differential Geometry is not just for mathematics majors, but encompasses techniques and ideas relevant to students in engineering and the sciences. Furthermore, the subject itself is not quantized. By this, I mean that there is a continuous spectrum of results that proceeds from those which depend on calculation alone to those whose proofs are quite abstract. In this way students gradually are transformed from calculators to thinkers. Into the mix of these ideas now comes the opportunity to visualize concepts and constructions through the use of computer algebra systems such as Maple and Mathematica. Indeed, it is often the case that the consequent visualization goes hand-in-hand with the understanding of the mathematics behind the computer construction. For instance, in Chapter 5, I use Maple to visualize geodesics on surfaces and this requires an understanding of the idea of solving a system of differential equations numerically and displaying the solution. Further, in this case, visualization is not an empty exercise in computer technology, but actually clarifies various phenomena such as the bound on geodesics due to the Clairaut relation. There are many other examples of the benefits of computer algebra systems to understanding concepts and solving xiii xiv Preface problems. In particular, the procedure for plotting geodesics can be modified to show equations of motion of particles constrained to surfaces. This is done in Chapter 7 along with describing procedures relevant to the calculus of variations and optimal control. At the end of Chapters 1,2, 3, 5 and 7 there are sections devoted to explaining how Maple fits into the framework of Differential Geometry. I have tried to make these sections a rather informal tutorial as opposed to just laying out procedures. This is both good and bad for the reader. The good comes from the little tips about pitfalls and ways to avoid them; the bad comes from my personal predilections and the simple fact that I am not a Maple expert. What you will find in this text is the sort of Maple that anyone can do. Also, I happen to think that Maple is easier for students to learn than Mathematica and so I use it here. If you prefer Mathematica, then you can, without too much trouble I think, translate my procedures from Maple into Mathematica. In spite of the use of computer algebra systems here, this text is traditional in the sense of approaching the subject from the point of view of the 19th century. What is different about this book is that a conscious effort has been made to include material that I feel science and math majors should know. Also, even when dealing with mathematical matters alone, I have always tried to keep some application, whether mathematical or not, in mind. In fact, I think this helps to show the boundaries between physics e. The book, as it now stands, is suitable for either a one-quarter or one-semester course in Differential Geometry as well as a full-year course. In the case of the latter, all chapters may be completed. In the case of the former, I would recommend choosing certain topics from Chapters and then allowing students to do projects, say, involving other parts. This carries students through the basic geometry of curves and surfaces while introducing various curvatures and applying virtually all of these ideas to study geodesics. My personal predilections would lead me to use Maple extensively to foster a certain geometric intuition. A second semester course could focus on the remainder of Chapter 5, Chapter 6 and Chapter 7 while saving Chapter 4 on minimal surfaces or Chapter 8 on higher dimensional geometry for projects. Students then will have seen Gauss-Bonnet, holonomy and a kind of recapitulation of geometry together with a touch of mechanics in terms of the Calculus of Variations. There are, of course, many alternative courses hidden within the book and I can only wish "good hunting" to all who search for them. Projects I think that doing projects offers students a chance to experience what it means to do research in mathematics. The mixture of abstract mathematics and its computer realization affords students the opportunity to conjecture and experiment much as they would do in the physical sciences. In Appendix C, I give five Preface xv suggestions for student projects in differential geometry. Certainly, experienced instructors will be able to point the way to other projects, but the ones suggested are mostly "self-contained" within the text. Groups of three or four students working together on projects such as these can truly go beyond their individual abilities to gain a profound understanding of some aspect of geometry in terms of proof, calculation and computer visualization. Prerequisites I have taught differential geometry to students e. But this surely is a minimum! While a touch of linear algebra is used in the book, I think it can be covered concurrently as long as a student has seen matrices. Having said this, I think a student can best appreciate the interconnections among mathematical subjects inherent in differential geometry when the student has had the experience of one or two upper-level courses. Book Features The reader should note two things about the layout ofthe book. First, the exercises are integrated into the text. While this may make them somewhat harder to find, it also makes them an essential part of the text. The reader should at least read the exercises when going through a chapter they are important. Secondly, I have chosen to number theorems, lemmas, examples, definitions and remarks in order as is usually done using LaTeX. To make it a bit easier to find specific examples, there is a list of examples with titles and the pages they are on in Appendix A. Elliptic Functions and Maple Note In recent years, I have become convinced of the utility of the elliptic functions in differential geometry and the calculus of variations, so I have included a simplified, straightforward introduction to these here. The main applications of elliptic functions presented here are the derivation of explicit parametrizations for unduloids and for the Mylar balloon. Such explicit parametrizations allow for the determination of differential geometric invariants such as Gauss curvature as well as an analysis of geodesics. Of course, part of this analysis involves Maple. These applications of elliptic functions are distillations of joint work with Ivailo Mladenov and I want to acknowledge that here with thanks to him for his insights and diligence concerning this work. Unfortunately, in going from Maple 9 to Maple 10, Maple developers introduced an error a misprint! Therefore, there may be slight differences between the Maple output as displayed in the text and what Maple 10 xvi Preface gives, but these differences are usually minor. These corrections are due to Alec Mihailovs. The rest of the procedures in the book work fine in Maple Thanks Since the publication of the First Edition, many people have sent me comments, suggestions and corrections. I have tried to take all of these into account in preparing the present MAA Edition, but sometimes this has proved to be impossible. One reason for this is that I want to keep the book at a level that is truly accessible to undergraduates. On the other hand, I have learned a great deal from all of the comments sent to me and, in some sense, this is the real payment for writing the book. Therefore, I want to acknowledge a few people who went beyond the call of duty to give me often extensive commentary. These folks are in alphabetical order! Thanks to all of you. Finally, the writing of this book would have been impossible without the help, advice and understanding of my wife Jan and daughter Kathy. For Users of Previous Editions The Maple work found in the present MAA Edition once again focusses on actually doing interesting things with computers rather than simply drawing pictures. Nevertheless, there are many more pictures of interesting phenomena in this edition. The pictures have all been created by me with Maple. In fact, by examining the Maple sections at the ends of chapters, it is usually pretty clear how all pictures were created. The version of Maple used for the Second Edition was Maple 8. The Maple work in the First Edition needed extensive revision to work with Maple 8 because Maple developers changed the way certain commands work. However, going from Maple 8 to Maple 10 has been much easier and the reader familiar with the Second Edition will have no trouble with the MAA Edition. Everything works the same. Should newer versions of Maple cause problems for the xvii

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Preface procedures in this book, look at my website for updates. One thing to pay attention to concerning this issue of Maple command changes is the following. Maple no longer supports the "linalg" package. Rather, Maple has moved to it package called "LinearAlgebra" and I have changed all Maple work in the book to reflect this. This should be stable for some time to come, no matter what new versions of Maple arise. Because of this, Maple plays an even more important role through its "dsolve" command and its ability to solve differential equations explicitly and numerically. Maple 8 to 9 Maple 9 appeared in Summer of and all commands and procedures originally written for Maple 8 work with one small exception. The following Maple 8 code defines a surface of revolution with functions g and h. The fix for Maple 9 is simple. Go straight to the definition of "surfrev". In the present MAA edition, the code has been modified to do just this, but if you are still using Maple 8, then use the code above. Note to Students Every student who takes a mathematics class wants to know what the real point of the course is. Often, courses proceed by going through a list of topics with accompanying results and proofs and, while the rationale for the ordering and presentation of topics may be apparent to the instructor, this is far from true for students. Books are really no different; authors get caught up in the "material" because they love their subject and want to show it off to students. Differential geometry is concerned with understanding shapes and their properties in terms of calculus. We do this in two main ways. We start by defining shapes using "formulas" called parametrizations and then we take derivatives and algebraically manipulate them to obtain new expressions that we show represent actual geometric entities. So, if we have geometry encoded in the algebra of parametrizations, then we can derive quantities telling us something about that geometry from calculus. The prime examples are the various curvatures which will be encountered in the book. Once we see how these special quantities arise from calculus, we can begin to tum the problem around by restricting the quantities in certain ways and asking what shapes have quantities satisfying these restrictions. For instance, once we know what curvature means, we can ask what plane curves have curvature functions that are constant functions. Since this is, in a sense, the reverse of simply calculating geometric quantities by differentiation, we should expect that "integration" arises here. More precisely, conditions we place on the geometric quantities give birth to differential equations whose solution sets "are" the shapes we are looking for. So differential geometry is intimately tied up with differential equations. Being able to handle separable differential equations and knowing a few tricks which can be learned along the way are usually sufficient.

Chapter 2 : John Oprea's Differential Geometry Book

Differential Geometry and its calendrierdelascience.com John Oprea. The Mathematical Association of America, 2nd edition, , xxi+ pages, ISBN

Tu 8 am Sequoia Office Hours: M2, W9, F2 Text: The final exam will be given on Monday, March 17, am in Solis It will cover all the material of the course. Solutions to Midterm 2 are in Soft Reserves. Goals of the Course- We will be learning the differential geometry of curves and surfaces. The idea is to study geometry using the ideas of calculus. A curve can be studied by looking at its tangent vectors. In one variable calculus, the tangent vectors are determined by the derivative and any two curves that are functions that have the sam derivative differ by a constant. We would like to approach curves in space using the same trick of taking derivatives. Is there a classification of curves in 3 space based on what the various derivatives are? We will answer this question in the first couple weeks of the course. We then go on to surfaces which are called two dimensional manifolds in differential geometry. An underlying theme will be the use of linear algebra to study geometric objects. It is too hard to study each object point by point, but if there are linear transformations or tangent vectors, we can use all the tools of linear algebra. Much of this theory generalizes to manifolds of arbitrary dimension, but this is too abstract for an intermediate level course. A byproduct of this study will be a brief excursion into topology also. This is a more conceptual course than it is a computational course. You will be introduced to many new concepts, with fewer examples to compute. It will be useful to keep track of the usefulness of each concept. Occasionally, there will be starred homework problems to prompt you to think about the significance of the problem. Mathematics is a participatory activity. Students are expected to attend class although it is at the early hour of 8 am. The student should actively join in the lectures for maximum understanding. This means that students will ask questions, talk with other students, present problems in class and debate the solutions and meanings of various problems. The student should not totally rely on lectures as the source of information. Students should also develop enough confidence so that they can read the book on their own. It is very important that you read the assigned material in advance of the lecture. This exercise will train you to eventually be able to read math books on your own. Understanding is obtained by doing many of the exercises. For this reason, homework will be heavily weighted in the determination of the grade. We will go over a number of exercises in class. Prerequisites- Math 20E and or consent of the instructor. It will be very helpful if students have taken 20F as well as Math Students who do not have this background will have to learn this material concurrently with the course. This course will require you to be familiar with the following concepts: There will be proofs so the ideas of Math will be useful. Students who have forgotten this material from Math 20C-F or Math should review this material during the first few weeks of the course. The first midterm will be given on Wednesday Feb 13, Midterm 2 will be given Wednesday March 12, Students should bring blue books, and a pencil. No calculators will be allowed. Grades The content of their questions in class will not be used to evaluate or grade the student. Only the homework, midterms and final will be used for that purpose. We will determine grades in the following way:

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Chapter 3 : Differential Geometry and Its Applications - John Oprea - Google Books

Differential geometry has a long, wonderful history it has found relevance in areas ranging from machinery design of the classification of four-manifolds to the creation of theories of nature's fundamental forces to the study of DNA.

Some adjustments may be necessary during the quarter. Professor Elham Izadi ; office: Gauss and mean curvatures, geodesics, parallel displacement, Gauss-Bonnet theorem. Additional references that could be useful: Banchoff and Stephen T. Elementary Differential Geometry second edition Theodore Shifrin: A First Course in Curves and Surfaces Lectures Differential Geometry is the study of geometry using the techniques of vector calculus and linear algebra. We will be using the material of Math 20E and 20F constantly, and you should review it. The central concept this quarter will be curvature: The material presented in lectures is important and is the main part of the course whether or not it is discussed in the textbook. The questions on the exams will test your understanding of concepts discussed in lectures. Your questions on any part of the material are always welcome in lectures or in sections. Please read the corresponding Section of the Syllabus before each lecture. Homework There will be weekly homework assignments which will be collected and graded. You are encouraged to work on these problem sets with a small group of other students in the class, but every student should write up his or her own solutions. Weekly homework will be due on Friday at Before you submit, please make sure to staple your work and have your name and ID number written clearly on top of the front page. No late homework will be accepted. However, the lowest homework grade will be dropped. Reading Please make sure to read the sections of the textbook corresponding to the assigned homework exercises; there will be some questions on the exams on the assigned reading whether or not it is discussed in the lecture please ask me or your TA for help with the assigned reading if you need help. Please read the corresponding sections of the Syllabus in advance of each lecture. Questions on any part of the material are always welcome in class or in sections. Wednesday December 7, As usual, the final exam is cumulative. Thursday October 20, in class Midterm 2: Thursday November 17, in class No make-up exams will be given please see the grading policy below in case you miss a midterm. No textbooks, notes, calculators, phones or electronic devices are allowed during exams. You do not need to bring anything other than a pen or pencil to the exam. We will not use blue books. Please ensure that you do not have a schedule conflict involving the final examination; you should not enroll in this class if you cannot take the final examination at its scheduled time. Grading Your final grade for the course will be the maximum of the following Homework: Regrade Requests You midterm exams will be returned to you in discussion section. If you wish to have the grader take a second look at your exam, please attach a note explaining your concern and return the exam to your TA. Regrade requests will not be considered once your exam leaves the room. The main issues are cheating and plagiarism. However, academic integrity also includes things like giving credit where credit is due listing your collaborators on homework assignments, noting books, webpages, or other resources containing information you used in solutions, etc.

Chapter 4 : Oprea Differential Geometry and Its Applications - PDF Free Download

Description. For undergraduate courses in Differential Geometry. Designed not just for the math major but for all students of science, this text provides an introduction to the basics of the calculus of variations and optimal control theory as well as differential geometry.

Chapter 5 : Differential Geometry and its Applications - Journal - Elsevier

This book studies the differential geometry of surfaces with the goal of helping students make the transition from the standard university curriculum to a type of mathematics that is a unified whole, by mixing geometry, calculus, linear *algebra, differential equations, complex variables, the calculus of variations, and notions from the sciences.*

Chapter 6 : Introduction To Differential Geometry

This is a very well-written text on modern differential geometry for undergraduates. The content of the book is similar to O'Neill's "Elementary Differential Geometry" (e.g. covariant derivatives, shape operators), but it's easier to read.

Chapter 7 : Differential Geometry and Its Applications

The point of this book is to mix together differential geometry, the calculus of variations and some applications (e.g. soap film formation, constrained particle motion, Foucault's pendulum) to see how geometry fits into science and mathematics.

Chapter 8 : "Differential Geometry and its Applications" by John F. Oprea

John Oprea begins Differential Geometry and Its Applications with the notion that differential geometry is the natural next course in the undergraduate mathematics sequence after linear algebra.

Chapter 9 : Differential Geometry and its Applications (Classroom Resource Materials) by John Oprea

- Differential Geometry With Applications To Mechanics And Physics Polya and Szego - Problems and Theorems in Mathematical Analysis 2 Michael Spivak - A Comprehensive Introduction to Differential Geometry, Vol. 4.