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Abstract Cortical activity is the product of interactions among neuronal populations. Macroscopic electrophysiological phenomena are generated by these interactions. In principle, the mechanisms of these interactions afford constraints on biologically plausible models of electrophysiological responses. In other words, the macroscopic features of cortical activity can be modelled in terms of the microscopic behaviour of neurons. An evoked response potential ERP is the mean electrical potential measured from an electrode on the scalp, in response to some event. The purpose of this paper is to outline a population density approach to modelling ERPs. We propose a biologically plausible model of neuronal activity that enables the estimation of physiologically meaningful parameters from electrophysiological data. The model encompasses four basic characteristics of neuronal activity and organization: This leads to a formulation of population dynamics in terms of the Fokker-Planck equation. The solution of this equation is the temporal evolution of a probability density over state-space, representing the distribution of an ensemble of trajectories. Each trajectory corresponds to the changing state of a neuron. Measurements can be modelled by taking expectations over this density, *e.* The key motivation behind our approach is that ERPs represent an average response over many neurons. This means it is sufficient to model the probability density over neurons, because this implicitly models their average state. Although the dynamics of each neuron can be highly stochastic, the dynamics of the density is not. This means we can use Bayesian inference and estimation tools that have already been established for deterministic systems. The potential importance of modelling density dynamics as opposed to more conventional neural mass models is that they include interactions among the moments of neuronal states *e.* Here, we formulate a population model, based on biologically informed model-neurons with spike-rate adaptation and synaptic dynamics. Neuronal sub-populations are coupled to form an observation model, with the aim of estimating and making inferences about coupling among sub-populations using real data. We approximate the time-dependent solution of the system using a bi-orthogonal set and first-order perturbation expansion. For didactic purposes, the model is developed first in the context of deterministic input, and then extended to include stochastic effects. The approach is demonstrated using synthetic data, where model parameters are identified using a Bayesian estimation scheme we have described previously.

Introduction Neuronal responses are the product of coupling among hierarchies of neuronal populations. Sensory information is encoded and propagated through the hierarchy depending on biophysical parameters that control this coupling. Because coupling can be modulated by experimental factors, estimates of coupling parameters provide a systematic way to parametrize experimentally induced responses, in terms of their causal structure. This paper is about estimating these parameters using a biologically informed model. Electroencephalography EEG is a non-invasive technique for measuring electrical activity generated by the brain. The electrical properties of nervous tissue derive from the electrochemical activity of coupled neurons that generate a distribution of current sources within the cortex, which can be estimated from multiple scalp electrode recordings Mattout et al. An interesting aspect of these electrical traces is the expression of large-scale coordinated patterns of electrical potential. There are two commonly used methods to characterize event-related changes in these signals: Particular characteristics of ERPs are associated with cognitive states, *e.* ERD is associated with an increase in processing information, *e.* A complementary strategy is to use a generative model of how data are caused, and estimate the model parameters that minimize the difference between real and generated data. This approach goes beyond associating particular activities with cognitive states to model the self-organization of neural systems during functional processing. Candidate models developed in theoretical neuroscience can be divided into mathematical and computational Dayan Computational models are concerned with how a computational device could implement a particular task, *e.* Both levels of analysis have produced compelling models, which speaks to the use of biologically and computationally informed forward or generative models in neuroimaging. We focus here on mathematical models. The model equations for a population are a set of nonlinear differential equations forming a closed

loop between the influence neuronal firing has on mean membrane potential and how this potential changes the consequent firing rate of a population. Usually, two operators are required: They are divided into lumped models Lopes da Silva et al. Frank and Frank et al. In doing this, they were able to infer changes in effective connectivity, defined as the influence one region exerts on another Aertsen et al. Analyses of effective connectivity in the neuroimaging community were first used with positron emission tomography and later with functional magnetic resonance imaging fMRI data Friston et al. The latter applications led to the development of dynamic causal modelling DCM Friston et al. DCM for neuroimaging data embodies organizational principles of cortical hierarchies and neurophysiological knowledge e. A principled way of incorporating these constraints is in the context of Bayesian estimation Friston et al. Furthermore, established Bayesian model comparison and selection techniques can be used to disambiguate different models and their implicit assumptions. The development of this methodology by David et al. These models explicitly show the effect of stochastic influences, e. This randomness is described probabilistically in terms of a probability density over trajectories through state space. The ensuing densities can be used to generate measurements, such as the mean firing rate or membrane potential of an average neuron within a population. In contrast, NMM models only account for the average neuronal state, and not for stochastic effects. Stochastic effects are known to be important for many phenomena, e. This equation has a long history in the physics of transport processes and has been applied to a wide range of physical phenomena, e. Brownian motion, chemical oscillations, laser physics and biological self-organization Haken , ; Kuramoto ; Risken This can be modelled by a deterministic equation in the form of a parabolic partial differential equation PDE. The Fokker-Planck formalism uses notions from mean-field theory, but is dynamic, and can model transitions from non-equilibrium to equilibrium states. ERPs represent the average response over millions of neurons, which means it is sufficient to model their population density to generate responses. Furthermore, the population dynamics entailed by the FPE are deterministic. This means Bayesian techniques that are already established for deterministic dynamical systems can be used to model electrophysiological responses. This cost could preclude a computationally efficient role in data analysis. The purpose of this paper was to assess the feasibility of using the FPE in a forward model of average neuronal responses. The model encompasses four basic characteristics of neuronal activity and organization; neurons are i dynamic units, ii driven by stochastic forces, iii organized into populations with similar biophysical properties and response characteristics and iv multiple populations interact to form functional networks. In the second section, we briefly review the Bayesian estimation scheme used in current DCMs Friston et al. In the third, we discuss features of the model and demonstrate the face-validity of the approach using simulated data. This involves inverting a population density model to estimate model parameters given synthetic data. Theory a A deterministic model neuron The response of a model neuron to input,  $s(t)$ , has a generic form, which can be represented by the differential equation.

## Chapter 2 : Stochastics and Dynamics - Wikipedia

*Stochastics and Dynamics (SD)* is an interdisciplinary journal published by World calendrierdelascience.com was founded in and covers "modeling, analyzing, quantifying and predicting stochastic phenomena in science and engineering from a dynamical system's point of view".

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Chapter 3 : Stochastic models of neuronal dynamics

*Introduction. A stochastic or random process can be defined as a collection of random variables that is indexed by some mathematical set, meaning that each random variable of the stochastic process is uniquely associated with an element in the set.*

This variable takes on a different set of values everytime we sample it. However, to be a useful quantity to describe the real world, this random variable should have well-defined statistical properties, which are hopefully experimentally accessible. In other words, all joint probability densities have time translation invariance. It is then related to the power spectral density of the process by Fourier transform, via the Wiener-Khinchine relations which are valid for stationary processes. Its characteristic decay time gives a measure of the noise "correlation time",  $\tau_c$ . Here the brackets denote averaging over an infinite ensemble of possible realizations of the noise, in the theoretical case, or a finite ensemble in an experimental or numerical setting. Such ensemble averages can be replaced by time averages if the system is ergodic. The integral of the autocorrelation function over all times is the noise intensity. Note also that the density can be time-dependent, some of its moments may not be defined, and the correlation function may depend on both its arguments, or one their difference if the system is stationary. Figure 1 shows different realizations of a noise process known as the Wiener process. Coupling the noise to the dynamics One of the difficulties with modeling noise is that we may not have access to the noise variable itself, but rather, to a state variable perturbed by one or more sources of noise. Thus, one may have to make assumptions about the nature of the noise and its coupling to the dynamical state variables. The accuracy of these assumptions can later be assessed by looking at the agreement between the predictions of the resulting model and the experimental data. Observation noise In this case, the dynamical system evolves deterministically, but the measurements on this system are contaminated by noise. While this is often an important source of noise, and the simplest to deal with mathematically, it is also the most boring form of noise: The deterministic evolution equation Eq. Additive noise A classification exists according to how the noise and dynamical variable interact. In other words, the noise is simply added to the deterministic part of the dynamics. Multiplicative noise Alternatively, one can have multiplicative noise, for which the coefficient of the noise depends on the value of one or many state variables. In such a case, the evolution equation would take the form: It is the time derivative of the Wiener process also known as Brownian motion  $W(t)$ ,  $\dot{W}(t)$ . Its intensity is the integral of the autocorrelation function, here equal to 1. This latter markov process begins at zero, and has Gaussian transition probabilities. Gaussian white noise is a good approximation to a colored noise process see below in the case where the characteristic time scales of the deterministic system are much larger than the noise correlation time. This is called the Quasi-White approximation. Ornstein-Uhlenbeck process The Ornstein-Uhlenbeck process OU was proposed to model the velocity of a particle executing Brownian motion its position is then obtained by integration. It is the only stationary Markovian process that is Gaussian and a diffusion process. Its realizations are continuous, and successive values are correlated exponentially. This latter property makes the OU process a "colored" noise. It is characterized by two parameters: Different scalings of the OU process are used in the literature. Its autocorrelation function is: For example, in Fig. It is also referred to as the random telegraph signal. In this case it is possible to obtain a linear equation which describes the time-evolution of the probability density. This equation is more complicated than the Fokker-Planck equation see below, but the stationary density can still be obtained analytically, at least for the case where there is only one dynamical variable. In contrast, no exact evolution equation for the probability density of the state variable can be obtained for the case of the OU noise. Also, there exists a limit of dichotomous noise that is white shot noise - which is a sequence of delta-function spikes - from which it is possible to transform to Gaussian white noise. Ito or Stratonovich calculus This distinction concerns stochastic differential equations that involve Gaussian white noise. Two main interpretations are used in the literature, known as the Ito and Stratonovich interpretations. It is important to establish what interpretation or "calculus" one assumes at the outset of an analysis, as this may influence the resulting form of the stochastic differential equation SDE. It will further affect the kind of calculus to use upon

making variable changes. The Stratonovich calculus obeys the usual laws of calculus such as for changes of variables, but this is not the case for the Ito calculus. Nevertheless it is possible to convert from one form of calculus to the other, and to restate an SDE in one form into the other as well as their corresponding Fokker-Planck equations - see below. The properties obtained with both calculi are identical when the Gaussian white noise is additive. Further, it is necessary to use a numerical integration method Kloeden and Platen that is compatible with the chosen calculus in order to match up simulation to theory. For example, the explicit Euler method is compatible with the Ito interpretation. In the following text the Ito calculus is used.

**Analysis of a stochastic dynamical system** One may be interested in the behavior of individual trajectories, features of which can be compared to those from experimental measurements, or in the evolution of probability densities. The SDE approach is concerned with the former, and involves either exact or approximate analytical solutions, or numerical solutions; the Fokker-Planck approach or more generally, the Chapman-Kolmogorov approach - see below focusses on time-dependent probability densities. The SDE approach can also be used to compute densities relevant to the latter approach. Such generators are in fact high-dimensional chaotic systems. Such algorithms must be "seeded", i. One of the results of the theory of Gaussian white noise is that the random term in Eq. Appropriate averages of the relevant properties can be computed over many realizations, each with a different seed. Fokker-Planck equation One can study how an ensemble of initial conditions, characterized by an initial density, propagates under the action of the SDE Eq. In one-dimension, this equation reads: Setting the left hand side of Eq. One can also solve the Fokker-Planck equation numerically, or adopt the Langevin approach, and estimate the density directly from realizations of the SDE. This is a statement of the fact that only the present state is necessary to compute the future state; the past is irrelevant for this computation. This Markov assumption is not strictly valid in any physical setting, where the immediate history will play a role in the future evolution see Gardiner ; Risken Nevertheless, the mathematical idealization that is the Markov process is useful for describing reality. The differential form of the Chapman-Kolmogorov equation can also be studied. For a one-dimensional system it reads multivariate forms can be found in Gardiner ; Risken The process then has continuous paths. Noise-induced states Noise-induced states are a nontrivial effect of noise. Their study requires the prior definition of the notion of a "state" in a stochastic sense, distinct from that of a "state variable". A stochastic state is the analogue of an attractor in a deterministic dynamical system. Specifically, it is the value of the dynamical variable for which the stationary probability distribution is a maximum. There can be more than one such state. These states may be the same as in the noiseless case. However, the positions and even the number of stochastic states may differ from the deterministic case. In the case where the number of stochastic states is larger than the number of stable deterministic fixed points, one speaks of the creation of stochastic states by noise. Examples across a variety of disciplines in the natural sciences can be found in the book by Horsthemke and Lefever, and in Schimansky-Geier et al. Generally noise reveals the presence of nearby bifurcations by producing behavior that is stereotyped for a given bifurcation Wiesenfeld It can produce stochastic versions of various deterministic phenomena such as phase locking Longtin and Chialvo, as in neurons and other excitable systems, in which also can create very long time scales, e. Noise as chaos As mentioned in the introduction, noise can have high-dimensional deterministic origins. In fact, a pseudo-random number generator is one such system operating in discrete time, i. Chaotic systems share many properties with noisy systems Lasota and Mackey, , such as their ability to synchronize Pikovsky et al. A recent review of the effect of chaotic dynamics as "deterministic brownian motion" on other dynamical systems can be found in Mackey and Tyran-Kaminska

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## Chapter 4 : Stochastic process - Wikipedia

*Stochastics and Dynamics | Citations: | This interdisciplinary journal is devoted to publishing high quality papers in modeling, analyzing, quantifying and predicting stochastic phenomena in.*

## Chapter 5 : Stochastics and Dynamics

*The fundamental problem of stochastic dynamics is to identify the essential characteristics of system (its state and evolution), and relate those to the input parameters of the system and initial data.*

## Chapter 6 : REU in Dynamics and Stochastics | Division of Applied Mathematics

*Stochastic Dynamics, Brownian Dynamics and Diffusion Stochastic Dynamics To this point, we have only discussed molecular dynamics (MD) as a method for simulating.*

## Chapter 7 : Stochastics and Dynamics: Asymptotic Problems

*Description In Stochastic Dynamics of Structures, Li and Chen present a unified view of the theory and techniques for stochastic dynamics analysis, prediction of reliability, and system control of structures within the innovative theoretical framework of physical stochastic systems.*

## Chapter 8 : Stochastic dynamical systems - Scholarpedia

*REU in Dynamics and Stochastics The Division of Applied Mathematics at Brown University will be hosting an REU Program in Dynamics and Stochastics during the summer The REU is part of an NSF-sponsored Research Training Group (RTG) on "Integrating Dynamics and Stochastics (IDYaS)."*

## Chapter 9 : Rental car stochastic dynamics - MetaSD

*(see [5]) or stochastic Volterra equations (see [6, 7]), the solution is in general not a semimartingale and it is only in special cases that the dynamics of such processes is known.*