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Chapter 1 : Finite Element Analysis of Solids and Fluids I | Mechanical Engineering | MIT OpenCourseWar

The extended finite element method (XFEM) is a numerical technique based on the generalized finite element method (GFEM) and the partition of unity method (PUM). It extends the classical finite element method by enriching the solution space for solutions to differential equations with discontinuous functions.

History[edit] The origin of finite method can be traced to the matrix analysis of structures [1] [2] where the concept of a displacement or stiffness matrix approach was introduced. Finite element concepts were developed based on engineering methods in s. The finite element method obtained its real impetus in the s and s by John Argyris , and co-workers; at the University of Stuttgart , by Ray W. Earlier books such as by Zienkiewicz [6] and more recent books such as by Yang [7] give comprehensive summary of developments in finite-element structural analysis. Implementing the method in software is described in the classic text by Smith, Griffiths and Margetts. This type of element is suitable for modeling cables, braces, trusses, beams, stiffeners, grids and frames. Straight elements usually have two nodes, one at each end, while curved elements will need at least three nodes including the end-nodes. The elements are positioned at the centroidal axis of the actual members. Two-dimensional elements that resist only in-plane forces by membrane action plane stress , plane strain , and plates that resist transverse loads by transverse shear and bending action plates and shells. They may have a variety of shapes such as flat or curved triangles and quadrilaterals. Nodes are usually placed at the element corners, and if needed for higher accuracy, additional nodes can be placed along the element edges or even within the element. The elements are positioned at the mid-surface of the actual layer thickness. Torus -shaped elements for axisymmetric problems such as membranes, thick plates, shells, and solids. The cross-section of the elements are similar to the previously described types: Three-dimensional elements for modeling 3-D solids such as machine components, dams , embankments or soil masses. Common element shapes include tetrahedrals and hexahedrals. Nodes are placed at the vertexes and possibly in the element faces or within the element. Element interconnection and displacement[edit] The elements are interconnected only at the exterior nodes, and altogether they should cover the entire domain as accurately as possible. Nodes will have nodal vector displacements or degrees of freedom which may include translations, rotations, and for special applications, higher order derivatives of displacements. When the nodes displace, they will drag the elements along in a certain manner dictated by the element formulation. In other words, displacements of any points in the element will be interpolated from the nodal displacements, and this is the main reason for the approximate nature of the solution. From the application point of view, it is important to model the system such that: Symmetry or anti-symmetry conditions are exploited in order to reduce the size of the model. Displacement compatibility, including any required discontinuity, is ensured at the nodes, and preferably, along the element edges as well, particularly when adjacent elements are of different types, material or thickness. Compatibility of displacements of many nodes can usually be imposed via constraint relations. The element mesh should be sufficiently fine in order to produce acceptable accuracy. To assess accuracy, the mesh is refined until the important results shows little change. For higher accuracy, the aspect ratio of the elements should be as close to unity as possible, and smaller elements are used over the parts of higher stress gradient. Proper support constraints are imposed with special attention paid to nodes on symmetry axes. Large scale commercial software packages often provide facilities for generating the mesh, and the graphical display of input and output, which greatly facilitate the verification of both input data and interpretation of the results. From elements, to system, to solution[edit] While the theory of FEM can be presented in different perspectives or emphases, its development for structural analysis follows the more traditional approach via the virtual work principle or the minimum total potential energy principle. The virtual work principle approach is more general as it is applicable to both linear and non-linear material behaviours. The virtual work method is an expression of conservation of energy: The principle of virtual displacements for the structural system expresses the mathematical identity of external and internal virtual work:

Chapter 2 : Finite Element Analysis Software | Autodesk

The finite element method is a systematic way to convert the functions in an infinite dimensional function space to first functions in a finite dimensional function space and then finally ordinary vectors (in a vector space) that are tractable with numerical methods.

For the vast majority of geometries and problems, these PDEs cannot be solved with analytical methods. Instead, an approximation of the equations can be constructed, typically based upon different types of discretizations. These discretization methods approximate the PDEs with numerical model equations, which can be solved using numerical methods. The solution to the numerical model equations are, in turn, an approximation of the real solution to the PDEs. The finite element method FEM is used to compute such approximations. Take, for example, a function u that may be the dependent variable in a PDE i. The function u can be approximated by a function uh using linear combinations of basis functions according to the following expressions: The figure below illustrates this principle for a 1D problem. Here, the linear basis functions have a value of 1 at their respective nodes and 0 at other nodes. In this case, there are seven elements along the portion of the x-axis, where the function u is defined i. The coefficients are denoted by u0 through u7. One of the benefits of using the finite element method is that it offers great freedom in the selection of discretization, both in the elements that may be used to discretize space and the basis functions. In the figure above, for example, the elements are uniformly distributed over the x-axis, although this does not have to be the case. Smaller elements in a region where the gradient of u is large could also have been applied, as highlighted below. Both of these figures show that the selected linear basis functions include very limited support nonzero only over a narrow interval and overlap along the x-axis. Depending on the problem at hand, other functions may be chosen instead of linear functions. Another benefit of the finite element method is that the theory is well developed. The reason for this is the close relationship between the numerical formulation and the weak formulation of the PDE problem see the section below. For instance, the theory provides useful error estimates, or bounds for the error, when the numerical model equations are solved on a computer. Looking back at the history of FEM, the usefulness of the method was first recognized at the start of the s by Richard Courant, a German-American mathematician. While Courant recognized its application to a range of problems, it took several decades before the approach was applied generally in fields outside of structural mechanics, becoming what it is today. Finite element discretization, stresses, and deformations of a wheel rim in a structural analysis. For example, conservation laws such as the law of conservation of energy, conservation of mass, and conservation of momentum can all be expressed as partial differential equations PDEs. Constitutive relations may also be used to express these laws in terms of variables like temperature, density, velocity, electric potential, and other dependent variables. Differential equations include expressions that determine a small change in a dependent variable with respect to a change in an independent variable x, y, z, t. This small change is also referred to as the derivative of the dependent variable with respect to the independent variable. Say there is a solid with time-varying temperature but negligible variations in space. In this case, the equation for conservation of internal thermal energy may result in an equation for the change of temperature, with a very small change in time, due to a heat source g: Temperature, T, is the dependent variable and time, t, is the independent variable. The function may describe a heat source that varies with temperature and time. The equation is a differential equation expressed in terms of the derivatives of one independent variable t. Such differential equations are known as ordinary differential equations ODEs. In some situations, knowing the temperature at a time t0, called an initial condition, allows for an analytical solution of Eq. Oftentimes, there are variations in time and space. The temperature in the solid at the positions closer to a heat source may, for instance, be slightly higher than elsewhere. Such variations further give rise to a heat flux between the different parts within the solid. In such cases, the conservation of energy can result in a heat transfer equation that expresses the changes in both time and spatial variables x , such as: For a Cartesian coordinate system, the divergence of q is defined as: When a differential equation is expressed in terms of the derivatives of more than one independent variable, it is referred to as a partial differential equation PDE , since each derivative may represent a change in one direction out of several possible directions. In addition to Eq. Such knowledge can be applied in the initial condition and boundary conditions for Eq. In many situations, PDEs cannot be resolved with analytical methods to give the value of the dependent variables at different times and positions. It may, for example, be very difficult or impossible to obtain an analytic expression such as: Rather than solving PDEs analytically, an alternative option is to search for approximate numerical solutions to solve the numerical model equations. The finite element method is exactly this type of method â€" a numerical method for the solution of PDEs. Similar to the thermal energy conservation referenced above, it is possible to derive the equations for the conservation of momentum and mass that form the basis for fluid dynamics. Further, the equations for electromagnetic fields and fluxes can be derived for space- and time-dependent problems, forming systems of PDEs. Basis Functions and Test Functions Assume that the temperature distribution in a heat sink is being studied, given by Eq. The boundary conditions at these boundaries then become: The outward unit normal vector to the boundary surface is denoted by n. Equations 10 to 13 describe the mathematical model for the heat sink, as shown below. The domain equation and boundary conditions for a mathematical model of a heat sink. The next step is to multiply both sides of Eq. A Hilbert space is an infinite-dimensional function space with functions of specific properties. It can be viewed as a collection of functions with certain nice properties, such that these functions can be conveniently manipulated in the same way as ordinary vectors in a vector space. For example, you can form linear combinations of functions in this collection the functions have a well-defined length referred to as norm and you can measure the angle between the functions, just like Euclidean vectors. Indeed, after applying the finite element method on these functions, they are simply converted to ordinary vectors. The finite element method is a systematic way to convert the functions in an infinite dimensional function space to first functions in a finite dimensional function space and then finally ordinary vectors in a vector space that are tractable with numerical methods. The weak formulation is obtained by requiring 14 to hold for all test functions in the test function space instead of Eq. A problem formulation based on Eq. In the so-called Galerkin method, it is assumed that the solution T belongs to the same Hilbert space as the test functions. The relations in 14 and 15 instead only require equality in an integral sense. For example, a discontinuity of a first derivative for the solution is perfectly allowed by the weak formulation since it does not hinder integration. It does, however, introduce a distribution for the second derivative that is not a function in the ordinary sense. As such, the requirement 10 does not make sense at the point of the discontinuity. A distribution can sometimes be integrated, making 14 well defined. It is possible to show that the weak formulation, together with boundary conditions 11 through 13 , is directly related to the solution from the pointwise formulation. And, for cases where the solution is differentiable enough i. This is the first step in the finite element formulation. With the weak formulation, it is possible to discretize the mathematical model equations to obtain the numerical model equations. The Galerkin method â€" one of the many possible finite element method formulations â€" can be used for discretization. First, the discretization implies looking for an approximate solution to Eq. Finite element discretization of the heat sink model from the earlier figure. Once the system is discretized and the boundary conditions are imposed, a system of equations is obtained according to the following expression: The right-hand side is a vector of the dimension 1 to n. If the source function is nonlinear with respect to temperature or if the heat transfer coefficient depends on temperature, then the equation system is also nonlinear and the vector b becomes a nonlinear function of the unknown coefficients Ti. One of the benefits of the finite element method is its ability to select test and basis functions. It is possible to select test and basis functions that are supported over a very small geometrical region. This implies that the integrals in Eq. The support of the test and basis functions is difficult to depict in 3D, but the 2D analogy can be visualized. Assume that there is a 2D geometrical domain and that linear functions of x and y are selected, each with a value of 1 at a point i, but zero at other points k. The next step is to discretize the 2D domain using triangles and depict how two basis functions test or shape functions could appear for two neighboring nodes i and j in a triangular mesh. Tent-shaped linear basis functions that have a value of 1 at the corresponding node and zero on all other nodes. Two base functions that share an element have a basis function overlap. Two neighboring basis functions share two triangular elements. As such, there is some overlap between the two basis functions, as shown above. These contributions form the coefficients for the unknown vector T that correspond to the diagonal components of the system matrix Ajj. Say the two basis functions are now a little further apart. These functions do not share elements but they have one element vertex in common. As the figure below indicates, they do not overlap. Two basis functions that share one element vertex but do not overlap in a 2D domain. When the basis functions overlap, the integrals in Eq. When there is no overlap, the integrals are zero and the contribution to the system matrix is therefore zero as well. This means that each equation in the system of equations for 17 for the nodes 1 to n only gets a few nonzero terms from neighboring nodes that share the same element. The system matrix A in Eq. The solution of the system of algebraic equations gives an approximation of the solution to the PDE. The denser the mesh, the closer the approximate solution gets to the actual solution. The finite element approximation of the temperature field in the heat sink. Time-Dependent Problems The thermal energy balance in the heat sink can be further defined for time-dependent cases.

Chapter 3 : Download [PDF] An Analysis Of The Finite Element Method â€“ Fodreport eBook

The Finite Element Analysis (FEA) is the simulation of any given physical phenomenon using the numerical technique called Finite Element Method (FEM). Engineers use it to reduce the number of physical prototypes and experiments and optimize components in their design phase to develop better products, faster.

The subdivision of a whole domain into simpler parts has several advantages: A typical work out of the method involves 1 dividing the domain of the problem into a collection of subdomains, with each subdomain represented by a set of element equations to the original problem, followed by 2 systematically recombining all sets of element equations into a global system of equations for the final calculation. The global system of equations has known solution techniques, and can be calculated from the initial values of the original problem to obtain a numerical answer. In the first step above, the element equations are simple equations that locally approximate the original complex equations to be studied, where the original equations are often partial differential equations PDE. To explain the approximation in this process, FEM is commonly introduced as a special case of Galerkin method. The process, in mathematical language, is to construct an integral of the inner product of the residual and the weight functions and set the integral to zero. In simple terms, it is a procedure that minimizes the error of approximation by fitting trial functions into the PDE. The residual is the error caused by the trial functions, and the weight functions are polynomial approximation functions that project the residual. The process eliminates all the spatial derivatives from the PDE, thus approximating the PDE locally with a set of ordinary differential equations for transient problems. These equation sets are the element equations. They are linear if the underlying PDE is linear, and vice versa. This spatial transformation includes appropriate orientation adjustments as applied in relation to the reference coordinate system. The process is often carried out by FEM software using coordinate data generated from the subdomains. FEA as applied in engineering is a computational tool for performing engineering analysis. It includes the use of mesh generation techniques for dividing a complex problem into small elements, as well as the use of software program coded with FEM algorithm. In applying FEA, the complex problem is usually a physical system with the underlying physics such as the Euler-Bernoulli beam equation , the heat equation , or the Navier-Stokes equations expressed in either PDE or integral equations , while the divided small elements of the complex problem represent different areas in the physical system. FEA is a good choice for analyzing problems over complicated domains like cars and oil pipelines , when the domain changes as during a solid state reaction with a moving boundary , when the desired precision varies over the entire domain, or when the solution lacks smoothness. FEA simulations provide a valuable resource as they remove multiple instances of creation and testing of hard prototypes for various high fidelity situations. Another example would be in numerical weather prediction , where it is more important to have accurate predictions over developing highly nonlinear phenomena such as tropical cyclones in the atmosphere, or eddies in the ocean rather than relatively calm areas. Colours indicate that the analyst has set material properties for each zone, in this case a conducting wire coil in orange; a ferromagnetic component perhaps iron in light blue; and air in grey. Although the geometry may seem simple, it would be very challenging to calculate the magnetic field for this setup without FEM software, using equations alone. FEM solution to the problem at left, involving a cylindrically shaped magnetic shield. The ferromagnetic cylindrical part is shielding the area inside the cylinder by diverting the magnetic field created by the coil rectangular area on the right. The color represents the amplitude of the magnetic flux density , as indicated by the scale in the inset legend, red being high amplitude. The area inside the cylinder is low amplitude dark blue, with widely spaced lines of magnetic flux , which suggests that the shield is performing as it was designed to. History[edit] While it is difficult to quote a date of the invention of the finite element method, the method originated from the need to solve complex elasticity and structural analysis problems in civil and aeronautical engineering. Its development can be traced back to the work by A. Hrennikoff [4] and R. Courant [5] in the early s. Another pioneer was Ioannis Argyris. In the USSR, the introduction of the practical application of the method is usually connected with name of Leonard Oganesyan. Feng proposed a systematic numerical method for solving partial differential equations. The method was called the finite difference method based on variation principle, which was another independent invention of the finite element method. Although the approaches used by these pioneers are different, they share one essential characteristic: The finite element method obtained its real impetus in the s and s by the developments of J. Argyris with co-workers at the University of Stuttgart , R. Clough with co-workers at UC Berkeley , O. Further impetus was provided in these years by available open source finite element software programs. A finite element method is characterized by a variational formulation , a discretization strategy, one or more solution algorithms and post-processing procedures. Examples of variational formulation are the Galerkin method , the discontinuous Galerkin method, mixed methods, etc. A discretization strategy is understood to mean a clearly defined set of procedures that cover a the creation of finite element meshes, b the definition of basis function on reference elements also called shape functions and c the mapping of reference elements onto the elements of the mesh. Examples of discretization strategies are the h-version, p-version , hp-version , x-FEM , isogeometric analysis , etc. Each discretization strategy has certain advantages and disadvantages. A reasonable criterion in selecting a discretization strategy is to realize nearly optimal performance for the broadest set of mathematical models in a particular model class. There are various numerical solution algorithms that can be classified into two broad categories; direct and iterative solvers. These algorithms are designed to exploit the sparsity of matrices that depend on the choices of variational formulation and discretization strategy. Postprocessing procedures are designed for the extraction of the data of interest from a finite element solution. In order to meet the requirements of solution verification, postprocessors need to provide for a posteriori error estimation in terms of the quantities of interest. When the errors of approximation are larger than what is considered acceptable then the discretization has to be changed either by an automated adaptive process or by action of the analyst. There are some very efficient postprocessors that provide for the realization of superconvergence. Illustrative problems P1 and P2[edit] We will demonstrate the finite element method using two sample problems from which the general method can be extrapolated. It is assumed that the reader is familiar with calculus and linear algebra. P1 is a one-dimensional problem P1.

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Chapter 4 : Finite Elements in Analysis and Design - Journal - Elsevier

The finite element method (FEM), or finite element analysis (FEA), is a computational technique used to obtain approximate solutions of boundary value problems in engineering.

Articles about Massively Open Online Classes MOOCs had been rocking the academic world at least gently , and it seemed that your writer had scarcely experimented with teaching methods. Particularly compelling was the fact that there already had been some successes reported with computer programming classes in the online format, especially as MOOCs. Finite Element Methods, with the centrality that computer programming has to the teaching of this topic, seemed an obvious candidate for experimentation in the online format. From there to the video lectures that you are about to view took nearly a year. I first had to take a detour through another subject, Continuum Physics, for which video lectures also are available, and whose recording in this format served as a trial run for the present series of lectures on Finite Element Methods. Here they are then, about 50 hours of lectures covering the material I normally teach in an introductory graduate class at University of Michigan. The treatment is mathematical, which is natural for a topic whose roots lie deep in functional analysis and variational calculus. It is not formal, however, because the main goal of these lectures is to turn the viewer into a competent developer of finite element code. We do spend time in rudimentary functional analysis, and variational calculus, but this is only to highlight the mathematical basis for the methods, which in turn explains why they work so well. Much of the success of the Finite Element Method as a computational framework lies in the rigor of its mathematical foundation, and this needs to be appreciated, even if only in the elementary manner presented here. A background in PDEs and, more importantly, linear algebra, is assumed, although the viewer will find that we develop all the relevant ideas that are needed. The development itself focuses on the classical forms of partial differential equations PDEs: At each stage, however, we make numerous connections to the physical phenomena represented by the PDEs. For clarity we begin with elliptic PDEs in one dimension linearized elasticity, steady state heat conduction and mass diffusion. We then move on to three dimensional elliptic PDEs in scalar unknowns heat conduction and mass diffusion , before ending the treatment of elliptic PDEs with three dimensional problems in vector unknowns linearized elasticity. Parabolic PDEs in three dimensions come next unsteady heat conduction and mass diffusion , and the lectures end with hyperbolic PDEs in three dimensions linear elastodynamics. Interspersed among the lectures are responses to questions that arose from a small group of graduate students and post-doctoral scholars who followed the lectures live. It is hoped that these lectures on Finite Element Methods will complement the series on Continuum Physics to provide a point of departure from which the seasoned researcher or advanced graduate student can embark on work in continuum computational physics. There are a number of people that I need to thank: Current research interests include:

Chapter 5 : Detailed Explanation of the Finite Element Method (FEM)

Finite Element Analysis or Finite Element Method (FEM) is a computer-based numerical method, for calculating the behavior and strength of engineering structures. It is also used to calculate deflection, vibration, buckling behavior, and stress.

Check new design of our homepage! It is also used to calculate deflection, vibration, buckling behavior, and stress. ScienceStruck Staff Last Updated: Jul 16, The finite element method FEM was introduced in the late sixties in the aerospace industry and it was applied in dentistry in the early seventies. It is a method, in which computerized numerical iteration technique is used. Using this technique, stresses and displacements can be determined using a predetermined model. FEM technique can be used to analyze small or even large-scale deflection, under applied or loading displacement. It is capable of analyzing plastic deformation of objects, which are permanently bent out of shape due to applied force; or elastic deformation. As inconceivably large number of calculations are required to analyze a large structure, a computer is needed. Finite element method is easily available to many disciplines and companies, because of the power and low cost of modern computers. Basic Concept Finite element method or FEM, solves a complex problem by redefining it as the summation of the solution of a series of interrelated simpler problems. In FEM, a complex structure is simplified by breaking it down into small elements. These elements are blocks, which form the structure. Each of the geometrical shape formed by these elements has a specific strain function. They can form shapes of triangle, tetrahedron, square etc. A relatively simple set of equations is used to describe the individual behavioral element. The whole structure is built using these set of elements. Behavior of the whole structure is described through an extremely large set of equations, which is obtained by the joining of equations describing the behavior of the individual elements. The computer is capable of solving this large set of simultaneous equations. The computer then extracts the behavior of the individual elements from the solution. After doing this, the computer gets the stress and deflection of all the parts of the structure. The strength of the structure is checked by comparing the stresses to its allowed values for the materials to be used. FEM enables the computer to evaluate a detailed and complex structure, during the planning of the structure. It also helps in increasing the rating of structures that were significantly over-designed. Generally two types of analysis are used in the industry, 2-D modeling and 3-D modeling. Whereas, 3-D modeling produces more accurate results, but it cannot effectively run only on normal computers. Numerous algorithms or functions can be inserted within each of these modeling schemes. These modeling schemes are responsible for the linear or non-linear behavior of the system. Linear systems are less complex and effective in determining elastic deformation. Many of the non-linear systems are capable of testing a material all the way to fracture, and they do account for plastic deformation. Applications of FEM It is used for the description of form changes in biological structures morphometrics , particularly in the area of growth and development. FEM and other related morphometric methods like the macro-element or the boundary integral equation method BIE are useful for assessment of complex shape changes. The knowledge of physiological values of alveolar stresses provides a guideline reference for the design of dental implants and it is also important for the understanding of stress related bone remodeling. It is useful with structures containing potentially complicated shapes like dental implants and inherent homogeneous material. It is useful for analysis of stresses produced in the periodontal ligament when subjected to orthodontic forces. It is also useful to study stress distribution in tooth in relation to different designs. It is used in the area of optimization of the design of dental restorations. It is used for investigation of stress distribution in tooth with cavity preparation. The type of predictive computer model described may be used to study the biomechanics of tooth movement, even though accurately assessing the effect of new appliance systems and materials without the need to go to animal or other less representative models. It is widely used in structural engineering. It is also used to predict and estimate the damages in electrical fields. It is also used in optimization of sheet metal blanking process FEM methods are widely applied in commercial fields because of its incredible precision in obtaining results. Its applications have also increased because of decreasing cost and increasing power of the computer.

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Chapter 6 : Finite Element Analysis - MATLAB & Simulink

() 14 Brief History - The term finite element was first coined by clough in In the early s, engineers used the method for approximate solutions of problems.

Chapter 7 : Finite element method in structural mechanics - Wikipedia

Introduction to finite element analysis. This free course is available to start right now. Review the full course description and key learning outcomes and create an account and enrol if you want a free statement of participation.

Chapter 8 : [PDF] Finite Element Method (Analysis) Books Collection Free Download â€“ EasyEngineering

Welcome to Finite Element Methods. The idea for an online version of Finite Element Methods first came a little more than a year ago. Articles about Massively Open Online Classes (MOOCs) had been rocking the academic world (at least gently), and it seemed that your writer had scarcely experimented with teaching methods.

Chapter 9 : Introduction to Finite Element Methods | Open Michigan

The Finite Element Method for Problems in Physics from University of Michigan. This course is an introduction to the finite element method as applicable to a range of problems in physics and engineering sciences.