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Chapter 1 : Differential Equations - Fourier Series

In mathematics, a Fourier series (/ Ëˆ f ÊŠr i eÉª, -i É™r /) is a way to represent a function as the sum of simple sine waves. More formally, it decomposes any periodic function or periodic signal into the weighted sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or, equivalently, complex exponentials).

Amplitude Relationship Now consider the phase part. This is because Fourier analysis uses cosines and sines. It is cosines, not the sines, which are the basic reference. More explanation of this is given in the slightly more mathematical part at the end of these notes. That is we had 0. Hence when we using an FFT to carry out the Fourier analysis then the separation between frequency points is 2Hz. This is a fundamental relationship. Selecting the FFT size, N, will dictate the effective duration of the signal being analysed. As we are dealing with the engineering analysis of signals measuring physical events it is clearly more sensible to ensure we can set our frequency spacing rather than the arbitrary choice of some FFT size which is not physically related to the problem in hand. That is DATS uses the natural default of physically meaningful quantities. This module does allow a choice of block size. They were specifically chosen that way. As noted earlier 0. Now, suppose we have a sine wave like the original 64Hz sine wave but at a frequency of 63Hz. This frequency is not an exact multiple of the frequency spacing. Visually it is very difficult to see any difference in the time domain but there is a distinct difference in the Fourier results. The graph below shows an expanded version of the result of an FFT of unit amplitude, zero phase, 63Hz sine wave. The values at 62Hz and 64Hz are almost identical, but they are not 0. That is the Fourier analysis is telling us we have a signal composed of multiple sine waves, the two principle ones being at 62 and 64Hz with half amplitudes of 0. In reality, we know we had a sine wave at 63Hz. If we overlay the modulus results at 63Hz and 64Hz then we note that the 63Hz curve is quite different in character to the 64Hz curve. The above results were obtained using an FFT algorithm. With the FFT the frequency spacing is a function of the signal length. Now given the speed of the modern PC then we may also use an original Direct Fourier Transform method. Choosing to analyse from 40Hz to 80Hz in 0. The peak value is 0. This leakage may be reduced by a suitable choice of data window. It is often better to think of this as the shape of the effective analysis filter. In this example, the data window used is a Bartlet rectangular type. Details of different data windows and their corresponding spectral window are discussed in a separate article. It is clear that with DFT and other techniques we can change the frequency spacing. But what is the frequency resolution? This is a large subject but we will give the essence. The clue is the shape of the spectral window as illustrated in Figure 7. A working definition of frequency resolution is the ability to separate two close frequency responses. Another common definition is the half power -3dB points of the spectral window. This is very similar to the half power points definition. ENBW is determined entirely by the shape of the data window used and the duration of the data used in the FFT processing. Signal Duration Effects If we have data taken over a longer period then the frequency spacing will be narrower. In many cases, this will assist the problem but if there is no exact match the same phenomenon will arise. Fourier analysis tells us the amplitude and phase of that set of cosines which have the same duration as the original signal. Suppose now we take a signal which again is composed of unit amplitude 64Hz sine wave and a 0. That is we now have a one second signal as shown below. Two sines joined The result of an FFT of these two joined signals is shown below. However, the half amplitudes are now 0. One interpretation of what the FFT is telling us is that there is a cosine wave at 64Hz of half amplitude 0. But we know that we had a half amplitude signal of 0. A closer look at the spectrum around 64Hz as shown below reveals that we have a large number of frequencies around 64Hz. This time they are 1Hz apart as we had one second of data. Their relative amplitudes and phases combine to double the amplitude at 64Hz over the first part and to cancel during the second part. The same of course happens in reverse around those frequencies close to Hz. FFT part of joined signals Another example is where a signal is extended by zeroes. Again the amplitude is reduced. In this case, the reduction is proportional to the percentage extension by zeroes. The important point to note is that the

Fourier analysis assumes that the sines and cosines last for the entire duration. Swept Sine Signal With a swept sine signal, theoretically, each frequency only lasts for an instant in time. A swept sine signal sweeping from 10Hz to Hz is shown below. Swept sine, unit amplitude,! The FFT of that signal shows an amplitude of about 0. The relationship between the spectrum level the amplitude and sweep rate of the original swept sine is not straightforward. A Fourier analysis shows the half amplitudes and phases of the constituent cosine waves that exist for the whole duration of that part of the signal that has been analysed. Although we have not discussed it, a Fourier analysed signal is invertible. That is if we have the Fourier analysis over the entire frequency range from zero to half sample rate then we may do an inverse Fourier transform to get back to the time signal. One point that arises from this is that if the signal being analysed has some random noise in it, then so does the Fourier transformed signal. Fourier analysis by itself does nothing to remove or minimise the effects of noise. Thus simple Fourier analysis is not suitable for random data, but it is for signals such as transients and complicated or simple periodic signals such as those generated by an engine running at a constant speed. They are discussed in another article. Where confusion occurs is that both analysis methods may use FFT algorithms. This is not to do with the objective of the analysis or its properties but rather with the efficiency of implementation. After all every analysis will use addition. That is just a mathematical operation and so, in that sense, is the use of an FFT. A Little Mathematics We will not go into all the mathematical niceties except to see that a Fourier series could be written in the forms below. In real and imaginary terms we have and in modulus and phase form as The above forms are a slightly unusual way of expressing the Fourier expansion. For instance is in degrees. More significantly the product is shown explicitly. Usually in an FFT then is expressed as.

Chapter 2 : What Is A Fourier Transform?

The amplitudes of the harmonics for this example drop off much more rapidly (in this case they go as 1/n 2 (which is faster than the 1/n decay seen in the pulse function Fourier Series (above)). Conceptually, this occurs because the triangle wave looks much more like the 1st harmonic, so the contributions of the higher harmonics are less.

The time domain signal used in the Fourier series is periodic and continuous. Figure shows several examples of continuous waveforms that repeat themselves from negative to positive infinity. Chapter 11 showed that periodic signals have a frequency spectrum consisting of harmonics. For instance, if the time domain repeats at hertz, the frequency spectrum will contain a first harmonic at hertz, a second harmonic at hertz, a third harmonic at hertz, and so forth. The first harmonic, i. This means that the frequency spectrum can be viewed in two ways: In other words, the frequencies between the harmonics can be thought of as having a value of zero, or simply not existing. The important point is that they do not contribute to forming the time domain signal. The Fourier series synthesis equation creates a continuous periodic signal with a fundamental frequency, f, by adding scaled cosine and sine waves with frequencies: The amplitudes of the cosine waves are held in the variables: In other words, the "a" and "b" coefficients are the real and imaginary parts of the frequency spectrum, respectively. In addition, the coefficient a0 is used to hold the DC value of the time domain waveform. This can be viewed as the amplitude of a cosine wave with zero frequency a constant value. Sometimes is grouped with the other "a" coefficients, but it is often handled separately because it requires special calculations. There is no b0 coefficient since a sine wave of zero frequency has a constant value of zero, and would be quite useless. The synthesis equation is written: Since the time domain signal is periodic, the sine and cosine wave correlation only needs to be evaluated over a single period, i. Selecting different limits makes the mathematics different, but the final answer is always the same. The Fourier series analysis equations are: Figure shows an example of calculating a Fourier series using these equations. The time domain signal being analyzed is a pulse train, a square wave with unequal high and low durations. The Fourier series coefficients can be found by evaluating Eq. First, we will find the DC component, a0: This result should make intuitive sense; the DC component is simply the average value of the signal. A similar analysis provides the "a" coefficients: The "b" coefficients are calculated in this same way; however, they all turn out to be zero. In other words, this waveform can be constructed using only cosine waves, with no sine waves being needed. The "a" and "b" coefficients will change if the time domain waveform is shifted left or right. Think about it this way. If the waveform is even i. This makes all of the "b" coefficients equal to zero. If the waveform if odd i. This results in the "a" coefficients being zero. To complete this example, imagine a pulse train existing in an electronic circuit, with a frequency of 1 kHz, an amplitude of one volt, and a duty cycle of 0. The table in Fig. Figure also shows the synthesis of the waveform using only the first fourteen of these harmonics. Even with this number of harmonics, the reconstruction is not very good. In mathematical jargon, the Fourier series converges very slowly. This is just another way of saying that sharp edges in the time domain waveform results in very high frequencies in the spectrum. Lastly, be sure and notice the overshoot at the sharp edges, i. An important application of the Fourier series is electronic frequency multiplication. Suppose you want to construct a very stable sine wave oscillator at MHz. This might be needed, for example, in a radio transmitter operating at this frequency. High stability calls for the circuit to be crystal controlled. That is, the frequency of the oscillator is determined by a resonating quartz crystal that is a part of the circuit. The problem is, quartz crystals only work to about 10 MHz. The solution is to build a crystal controlled oscillator operating somewhere between 1 and 10 MHz, and then multiply the frequency to whatever you need. This is accomplished by distorting the sine wave, such as by clipping the peaks with a diode, or running the waveform through a squaring circuit. The harmonics in the distorted waveform are then isolated with band-pass filters. This allows the frequency to be doubled, tripled, or multiplied by even higher integers numbers. The most common technique is to use sequential stages of doublers and triplers to generate the required frequency multiplication, rather than just a single stage. The Fourier series is important to this type of design because it describes the amplitude of the multiplied signal, depending on the type of distortion and harmonic selected.

Chapter 3 : Differential Equations - Fourier Cosine Series

The Fourier Transform Consider the Fourier coefficients. Let's define a function F(m) that incorporates both cosine and sine series coefficients, with the sine.

My odd function means that on the left side of 0, I get the negative of what I have on the right side of 0. F at minus x is minus f of x. The cosine function is even, and we will have no cosines here. All the integrals that involve cosines will tell us 0 for the coefficients AN. So you see that I chose a simple odd function, minus 1 or 1, which would give a square wave if I continue it on. It will go down, up, down, up in a square wave pattern. When I multiply them, I have an even function. And the integral from minus pi to 0 is just the same as the integral from 0 to pi. But my function on 0 to pi is 1. We can do this. Now I have to put in pi and 0 and put in the limits of integration and get the answer. So what do I get? I get 2 over pi. For k equal 1, I think I get-- so k is 1, the denominator will be 1, and I think the numerator is 2. Yes, when k is 0, I get yeah. When k is 1, I get 2. When k is 2, so this is 4 over pi, I figured out as the first coefficient. The coefficient b1 is 4 over pi. The coefficient b2, now if I take k equal to 2, I have a 2 down below. But above, I have a 0 because the cosine of 2 pi is the same as the cosine of 0. Now I go to k equals 3. So the k equals 3 will come down here. I get another 2. Good if you do these. And when k is 4, I get a 0 again. You see the pattern? The pattern for the integrals is the k is going 1, 2, 3, 4, 5. So I see that now for this function, which is better than the delta function also. I see some decay, some slow decay, in the Fourier coefficients. This factor k is growing so the numbers are going to 0, but not very fast. Because my function is not very smooth. I might as well take that 4 over pi times 1. There is an odd function. Sort of a repeating ramp function. And I see that function is even. And what does even mean? That means that my function at minus-- there is minus x-- is the same as the value at x. And what that means for a Fourier series is cosine. Even functions only have cosine terms. So this is this ramp, this repeating ramp function, is going to be 4 over pi. But why should I do that when I can just integrate? The integral of sine 3x is a cosine 3x over 3. And similarly here, when I integrate sine 5x I get cos 5x with a 5. And then I already had one 5, so 5 squared. So there you go. So there is a constant term, the average value, that a0. I think probably its average is about pi over 2, right? So let me sneak in the constant term here. The ramp is, I think I have a constant term is pi over 2. It would come from the formula and those-- well, what do you see now? You see a faster drop off. This function has corners. This function has jumps. So a jump is one level more rough, more word noisy than a ramp function. The smoother function has faster decay. Smooth-- let me write those words-- smooth function connects with faster decay. Faster drop off of the Fourier coefficient. It means that the Fourier series is much more useful. Fourier series is really terrific for functions that are smooth because then you only need to keep a few terms. For functions that have jumps or delta functions, you have to keep many, many terms and the Fourier series calculation is much more difficult. We learned something about integrating and taking the derivative so let me end with just two basic rules. So the rule for derivatives. And the second will be the rule for shift. You know that when I change x to x minus d, all that does is shift the graph by a distance d. That should do something nice to its Fourier coefficient. This would be a good time. Suppose start is f of x equals the sum of ck, a complex coefficient e to the ikx, the complex exponential. So I have a Fourier series. Well, just take the derivative. And here I see that the Fourier coefficient for a shifted function-- so the ck was a Fourier coefficient for f. When I shift f, it multiplies that coefficient by a phase change. Those are two would rules that show why you can use Fourier series in differential equations and in difference equations.

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Chapter 4 : The Fourier Series

WHY Fourier Transform? If a function f (t) is not a periodic and is defined on an infinite interval, we cannot represent it by Fourier series. It may be possible, however, to consider the function to be periodic with an infinite period.

Unfortunately, the meaning is buried within dense equations: What does the Fourier Transform do? Given a smoothie, it finds the recipe. Run the smoothie through filters to extract each ingredient. Recipes are easier to analyze, compare, and modify than the smoothie itself. How do we get the smoothie back? Time for the equations? From Smoothie to Recipe A math transformation is a change of perspective. The Fourier Transform changes our perspective from consumer to producer, turning What do I have? Well, recipes are great descriptions of drinks. A recipe is more easily categorized, compared, and modified than the object itself. Well, imagine you had a few filters lying around: Pour through the "banana" filter. Pour through the "orange" filter. Pour through the "milk" filter. Pour through the "water" filter. We can reverse-engineer the recipe by filtering each ingredient. Filters must be independent. The banana filter needs to capture bananas, and nothing else. Adding more oranges should never affect the banana reading. Filters must be complete. Our collection of filters must catch every possible ingredient. Ingredients must be combine-able. Smoothies can be separated and re-combined without issue A cookie? The ingredients, when separated and combined in any order, must make the same result. What if any signal could be filtered into a bunch of circular paths? This concept is mind-blowing, and poor Joseph Fourier had his idea rejected at first. Really Joe, even a staircase pattern can be made from circles? And despite decades of debate in the math community, we expect students to internalize the idea without issue. The Fourier Transform finds the recipe for a signal, like our smoothie process: Start with a time-based signal Apply filters to measure each possible "circular ingredient" Collect the full recipe, listing the amount of each "circular ingredient" Stop. The crackle of random noise can be removed. Maybe similar "sound recipes" can be compared music recognition services compare recipes, not the raw audio clips. If computer data can be represented with oscillating patterns, perhaps the least-important ones can be ignored. If a radio wave is our signal, we can use filters to listen to a particular channel. In the smoothie world, imagine each person paid attention to a different ingredient: Adam looks for apples, Bob looks for bananas, and Charlie gets cauliflower sorry bud. Think With Circles, Not Just Sinusoids One of my giant confusions was separating the definitions of "sinusoid" and "circle". A "circle" is a round, 2d pattern you probably know. If you enjoy using dollar words to describe cent ideas, you might call a circular path a "complex sinusoid". Labeling a circular path as a "complex sinusoid" is like describing a word as a "multi-letter". You zoomed into the wrong level of detail. Words are about concepts, not the letters they can be split into! Must we use imaginary exponents to move in a circle? What should I say? How big is the circle? Phase angle, where 0 degrees is the x-axis I could say "2-inch radius, start at 45 degrees, 1 circle per second, go! After half a second, we should each be pointing to: We can even combine paths: The combined position of all the cycles is our signal, just like the combined flavor of all the ingredients is our smoothie. The magnitude of each cycle is listed in order, starting at 0Hz. The blue graph measures the real part of the cycle. Another lovely math confusion: You can mentally rotate the circle 90 degrees if you like. The time points are spaced at the fastest frequency. The time values [1 -1] shows the amplitude at these equally-spaced intervals. The little motorcars are getting wild: Try toggling the green checkbox to see the final result clearly. The combined "flavor" is a sway that starts at the max and dips low for the rest of the interval. The yellow dots are when we actually measure the signal. In this case, cycles [0 1 1] generate the time values [2 -1 -1], which starts at the max 2 and dips low This is a shifted version of [0 1]. On the time side we get [. The Fourier Transform finds the set of cycle speeds, amplitudes and phases to match any time signal. Our signal becomes an abstract notion that we consider as "observations in the time domain" or "ingredients in the frequency domain". It behaves exactly as we need at the equally-spaced moments we asked for. Our cycle ingredients must start aligned at the max value, 4 and then "explode outwards", each cycle with partners that cancel it in the future. At time 0, the first instant, every cycle ingredient is at its max. Ignoring the other time points, 4? Imagine a constellation of points moving around the circle. Time 0 1 2 3 0Hz: When our cycle is 4 units long, cycle speeds a half-cycle apart 2 units will either be lined up difference of 0, 4, 8â€¦ or on opposite sides difference of 2, 6, 10â€¦. All cycles at their max total of 4 Time 1: The net is 0. The total is still 0. All cycles line up. When every cycle has equal power and 0 phase, we start aligned and cancel afterwards. Try [1 1], [1 1 1], [1 1 1 1] and notice the signals we generate: In my head, I label these signals as "time spikes": Can we change our spike to 0 4 0 0? Imagine a race with 4 runners. Normal races have everyone lined up at the starting line, the 4 0 0 0 time pattern. What if we want everyone to finish at the same time? Just move people forward or backwards by the appropriate distance. Maybe granny can start 2 feet in front of the finish line, Usain Bolt can start m back, and they can cross the tape holding hands. Phase shifts, the starting angle, are delays in the cycle universe. Discovering The Full Transform The big insight: If we merge the recipes for each time spike, we should get the recipe for the full signal. The Fourier Transform builds the recipe frequency-by-frequency: Separate the full signal a b c d into "time spikes": We need to offset each spike with a phase delay the angle for a "1 second delay" depends on the frequency. We can then loop through every frequency to get the full transform. The raw equations for the Fourier Transform just say "add the complex numbers". Many programming languages cannot handle complex numbers directly, so you convert everything to rectangular coordinates and add those. Onward This was my most challenging article yet. I was constantly bumping into the edge of my knowledge. That discomfort led me around the web to build my intuition. Imagine spinning your signal in a centrifuge and checking for a bias. I have a correction:

Chapter 5 : Aperiodic Functions

This version of the Fourier series is called the exponential Fourier series and is generally easier to obtain because only one set of coefficients needs to be evaluated. Example of Rectangular Wave As an example, let us find the exponential series for the following rectangular wave, given by.

Chapter 6 : Exponential Fourier Series with Solved Example | Electrical Academia

Throughout this section we will work exclusively with the Exponential Fourier Series (which will lead to the Fourier Transform). Behavior as T Increases In the previous docum e nt, the Fourier Series of the pulse function was derived and discussed.

Chapter 7 : An Interactive Guide To The Fourier Transform â€“ BetterExplained

The Fourier Series breaks down a periodic function into the sum of sinusoidal functions. It is the Fourier Transform for periodic functions. It is the Fourier Transform for periodic functions. To start the analysis of Fourier Series, let's define periodic functions.

Chapter 8 : From Fourier Series to Fourier Transform

Fourier series is really terrific for functions that are smooth because then you only need to keep a few terms. For functions that have jumps or delta functions, you have to keep many, many terms and the Fourier series calculation is much more difficult.

Chapter 9 : Fourier Transform and Inverse Fourier Transform with Examples and Solutions | Electrical Aca

The Fourier Transform: Examples, Properties, Common Pairs Constant Functions Spatial Domain Frequency Domain

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f(t) F (u) 1 (u) a a (u) The Fourier Transform: Examples, Properties, Common Pairs.