DOWNLOAD PDF ORDINARY DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENT

Chapter 1 : Constant Coefficients

In this introductory course on Ordinary Differential Equations, we first provide basic terminologies on the theory of differential equations and then proceed to methods of solving various types of ordinary differential equations. We handle first order differential equations and then second order.

Theorem A above says that the general solution of this equation is the general linear combination of any two linearly independent solutions. So how are these two linearly independent solutions found? The following example will illustrate the fundamental idea. This quadratic polynomial equation can be solved by factoring: Since these functions are linearly independent neither is a constant multiple of the other , Theorem A says that the general linear combination is the general solution of the differential equation. Once the roots of this auxiliary polynomial equation are found, you can immediately write down the general solution of the given differential equation. There are exactly three cases to consider. The discriminant is positive. In this case, the roots are real and distinct. The discriminant is zero. In this case, the roots are real and identical; that is, the polynomial equation has a double repeated root. The discriminant is negative. Determine the roots of this quadratic equation, and then, depending on whether the roots fall into Case 1, Case 2, or Case 3, write the general solution of the differential equation according to the form given for that Case. The auxiliary polynomial equation is whose roots are real and distinct: This problem falls into Case 1, so the general solution of the differential equation is Example 3: The auxiliary polynomial equation is which has a double root: This problem falls into Case 2, so the general solution of the differential equation is Example 4: The auxiliary quadratic equation is which has distinct conjugate complex roots: This problem falls into Case 3, so the general solution of the differential equation is Example 5: Solve the IVP First, rewrite the differential equation in standard form: Next, form the auxiliary polynomial equation and determine its roots:

Chapter 2 : Constant-Coefficient Linear Homogeneous ODE

A linear differential equation or a system of linear equations such that the associated homogeneous equations have constant coefficients may be solved by quadrature (mathematics), which means that the solutions may be expressed in terms of integrals. This is also true for a linear equation of order one, with non-constant coefficients.

A solution defined on all of R is called a global solution. A general solution of an nth-order equation is a solution containing n arbitrary independent constants of integration. A valuable but little-known work on the subject is that of Houtain Darboux starting in was a leader in the theory, and in the geometric interpretation of these solutions he opened a field worked by various writers, notable ones being Casorati and Cayley. To the latter is due the theory of singular solutions of differential equations of the first order as accepted circa Reduction to quadratures[edit] The primitive attempt in dealing with differential equations had in view a reduction to quadratures. As it had been the hope of eighteenth-century algebraists to find a method for solving the general equation of the nth degree, so it was the hope of analysts to find a general method for integrating any differential equation. Gauss showed, however, that the differential equation meets its limitations very soon unless complex numbers are introduced. Hence, analysts began to substitute the study of functions, thus opening a new and fertile field. Cauchy was the first to appreciate the importance of this view. Thereafter, the real question was to be not whether a solution is possible by means of known functions or their integrals but whether a given differential equation suffices for the definition of a function of the independent variable or variables, and, if so, what are the characteristic properties of this function. Collet was a prominent contributor beginning in , although his method for integrating a non-linear system was communicated to Bertrand in Clebsch attacked the theory along lines parallel to those followed in his theory of Abelian integrals. He showed that the integration theories of the older mathematicians can, by the introduction of what are now called Lie groups , be referred to a common source, and that ordinary differential equations that admit the same infinitesimal transformations present comparable difficulties of integration. He also emphasized the subject of transformations of contact. The theory has applications to both ordinary and partial differential equations. Symmetry methods have been recognized to study differential equations, arising in mathematics, physics, engineering, and many other disciplines. Sturmâ€"Liouville theory Sturmâ€"Liouville theory is a theory of a special type of second order linear ordinary differential equations. Their solutions are based on eigenvalues and corresponding eigenfunctions of linear operators defined in terms of second-order homogeneous linear equations. Liouville , who studied such problems in the mids. The interesting fact about regular SLPs is that they have an infinite number of eigenvalues, and the corresponding eigenfunctions form a complete, orthogonal set, which makes orthogonal expansions possible. This is a key idea in applied mathematics, physics, and engineering. Existence and uniqueness of solutions[edit] There are several theorems that establish existence and uniqueness of solutions to initial value problems involving ODEs both locally and globally. The two main theorems are Theorem.

Chapter 3 : How to Solve Homogeneous Linear Differential Equations with Constant Coefficients

This equation implies that the solution is a function whose derivatives keep the same form as the function itself and do not explicitly contain the independent variable, since constant coefficients are not capable of correcting any irregular formats or extra variables.

I fell into the trap that many web developers fall into. I knew what was in the menus and so clearly all the users would as well It was appearing that many new users were not aware of the Practice Problems on the site so I added a set of links at the top to allow for easy switching between the Notes, Practice Problems and Assignment Problems. They will only appear on the class pages which have Practice and Assignment problems. The links should stay at the top as you scroll through the page. Paul November 7, Mobile Notice You appear to be on a device with a "narrow" screen width i. Due to the nature of the mathematics on this site it is best views in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device should be able to scroll to see them and some of the menu items will be cut off due to the narrow screen width. Basic Concepts In this chapter we will be looking exclusively at linear second order differential equations. The most general linear second order differential equation is in the form. In the case where we assume constant coefficients we will use the following differential equation. Here is the general constant coefficient, homogeneous, linear, second order differential equation. This example will lead us to a very important fact that we will use in every problem from this point on. The example will also give us clues into how to go about solving these in general. We need functions whose second derivative is 9 times the original function. One of the first functions that I can think of that comes back to itself after two derivatives is an exponential function and with proper exponents the 9 will get taken care of as well. So, it looks like the following two functions are solutions. These two functions are not the only solutions to the differential equation however. Any of the following are also solutions to the differential equation. This example leads us to a very important fact that we will use in practically every problem in this chapter. This will work for any linear homogeneous differential equation. Since we have two constants it makes sense, hopefully, that we will need two equations, or conditions, to find them. We do give a brief introduction to boundary values in a later chapter if you are interested in seeing how they work and some of the issues that arise when working with boundary values. Another way to find the constants would be to specify the value of the solution and its derivative at a particular point. As with the first order differential equations these will be called initial conditions. Example 2 Solve the following IVP. At this point, please just believe this. You will be able to verify this for yourself in a couple of sections. This means that we need the derivative of the solution. For a rare few differential equations we can do this. However, for the vast majority of the second order differential equations out there we will be unable to do this. So, we would like a method for arriving at the two solutions we will need in order to form a general solution that will work for any linear, constant coefficient, second order homogeneous differential equation. This is easier than it might initially look. We will use the solutions we found in the first example as a guide. The important idea here is to get the exponential function. Once we have that we can add on constants to our hearts content. Okay, so how do we use this to find solutions to a linear, constant coefficient, second order homogeneous differential equation? Once we have these two roots we have two solutions to the differential equation. First write down the characteristic equation for this differential equation and solve it. We have three cases that we need to look at and this will be addressed differently in each of these cases. So, what are the cases? The roots will have three possible forms.

Chapter 4 : Linear Homogeneous Ordinary Differential Equations with Constant Coefficients

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Actually, I found that source is of considerable difficulty. And, in general, it is. Examples would be the RC circuit, radioactive decay, stuff like that. So, this is not a universal utility. But I thought that that form of writing it was a sufficient utility to make a special case, and I emphasize it very heavily in the nodes. And, this form will be good enough, the y prime. When you solve it, let me remind you how the solutions look, because that explains the terminology. So, you could either write it this way, where this is somewhat vague, or you could make it definite by putting a zero here and a t there, and change the dummy variable inside according to the way the notes tell you to do it. This term stays some sort of function. And this, which disappears, gets smaller and smaller as time goes on, is therefore called the transient because it disappears at the time increases to infinity. So, this part uses the initial condition, uses the initial value. The starting value appears in this term. This one is just some function. Well, what do the other guys look like? Well, the steady-state solution has this starting point. Other solutions can have any of these other starting points. But, we know that as time goes on, they must approach it because this term represents the difference between the solution and the steady-state solution. So, this term is going to zero. And therefore, whatever these guys do to start out with, after a while they must follow the steady-state solution more and more closely. They must, in short, be asymptotic to it. So, the solutions to any equation of that form will look like this. Up here, maybe it started at Now, but something follows from that picture. Which is the steady-state solution? What, in short, is so special about this green curve? All these other white solution curves have that same property, the same property that all the other white curves and the green curve, too, are trying to get close to them. In other words, there is nothing special about the green curve. Now, this produces vagueness. You talk about the steady-state solution; which one are you talking about? I have no answer to that; the usual answer is whichever one looks simplest. Normally, the one that will look simplest is the one where c is zero. But, if this is a peculiar function, it might be that for some other value of c, you get an even simpler expression. So, the steady-state solution: Pick the value of c, which gives you the simplest answer. The input is the q of t. In other words, it seems rather peculiar. But the input is the right-hand side of the equation of the differential equation. The external water bath at temperature T external, the internal thing here, the problem is, given this function, the external water bath temperature is driving, so to speak, the temperature of the inside. And therefore, the input is the temperature of the water bath. Anyone might be willing to say, yeah, you are inputting the value of the temperature here. This thing, this plus the water bath, is a little system. And the response of the system, i. So, the input is q of t, and the response of the system is the solution to the differential equation. By the way, this is often handled, I mean, how would you handle this to get rid of a k? Well, divide through by k. So, this equation is often, in the literature, written this way: But I will continue to call it q e. So, in other words, and this part this is just, frankly, called the input. So, this is also a way of writing the equation. Well, normally it means any solution, or in other words, the steady-state solution. Now, notice that terminology only makes sense if k is positive. So, this assumes definitely that k has to be greater than zero. Whether they are physical inputs or nonphysical inputs, if the input q of t produces the response, y of t, and q two of t produces the response, y two of t, -- then a simple calculation with the differential equation shows you that by, so to speak, adding, that the sum of these two, I stated it very generally in the notes but it corresponds, we will have as the response y1, the steady-state response y1 plus y2, and a constant times y1. But essentially, it uses the fact that the equation is linear. And, something like this would not, in any sense, be true if the equation, for example, had here a y squared instead of t. So, if you like, k here. So, the q e is cosine omega t. That was the physical input. And, omega, as you know, is, you have to be careful when you use the word frequency. I assume you got this from physics class all last semester. Instead of giving a long explanation, the end of the second page of the notes just gives you a reference list of what you are expected to know for So, think of it as something to refer back to if you have forgotten. This cosine omega t is going up and down right? So, a complete oscillation as it goes down and then returns to where it started. Okay, so the number of complete oscillations in how much time, well, in two pi, in the distance, two pi on the t-axis in the interval of length two pi because, for example, if omega is one, cosine t takes two pi to repeat itself, right? If omega were two, it would repeat itself. It would make two complete oscillations in the interval, two pi. In other words, solve the differential equation. The input was in green, maybe, but I do remember that the response was in yellow. I think I remember that. So, find the response, yellow, and the input was, what color was it, green? To use complex numbers, what you do is complexification of the problem. And therefore, try to introduce, try to change the trigonometric functions into complex exponentials, simply because the work will be easier to do. But, I will throw at the imaginary part, too, since at one point we will need it. Now, I have a problem because y, here, in this equation, y means the real function which solves that problem. I therefore cannot continue to call this y because I want y to be a real function. I have to change its name. Since this is complex function on the right-hand side, I will have to expect a complex solution to the differential equation. So, y tilda is the complex solution. And now, what I say is, so, solve it. Find this complex solution. And then I say, all you have to do is take the real part of that, and that will answer the original problem. It will be something different, but that part of it, the real part will solve the original problem, the original, real, ODE. Now, you will say, you expect us to believe that? Well, yes, in fact. It only takes a line or two of standard work with differentiation. It just amounts to separating real and imaginary parts. Well, just use integrating factors and just do it. So, the integrating factor will be e to the kt is the integrating factor. It will be k times e to the power i times omega t plus k. So, y tilda e to the kt is k divided by, now I integrate this, so it essentially reproduces itself, except you have to put down on the bottom k plus i omega. This is never scary. If I multiply this by e to the negative kt, then that just gets rid of the k that I put in, and left back with e to the i omega t. So, that side is easy. All that is left is e to the i omega t. And you scale it. And, what does that produce? One divided by one plus i times omega over k. Okay, now, what I have to do now is take the real part.

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Chapter 5 : Search second order differential equation with constant coefficients examples - GenYoutube

Ch Second Order Linear Homogeneous Equations with Constant Coefficients A second order ordinary differential equation has the general form where f is some given function.

A constant-coefficient homogeneous second-order ode can be put in the form where p and q are constants. Our goal is to find two linearly independent solutions of the ode. We must find r. This is considered a degenerate case and is neglected. Hence, r must satisfy the equation F r is called the characteristic polynomial. Solving the original ode is reduced to solving an algebraic equation. Assuming the coefficients p and q are real numbers, there are three cases to consider: Characteristic polynomial has distinct roots Characteristic polynomial has a double root Characteristic polynomial has complex conjugate roots Characteristic Polynomial has Distinct Roots Suppose that the characteristic polynomial has two distinct, real roots call them a and b. Then exp at and exp bt are linearly independent solutions to the original ode and the general solution to the ode is: Consider the following example: Characteristic Polynomial has a Double Root Suppose that the characteristic polynomial has a double root call it a. We need a second linearly independent solution to the ode to get the general solution. Using the technique of reduction of order , it can be shown that texp at is also a solution of the ode. The general solution is Consider the following example: These are distinct roots, so the the general solution can be written: The problem with writing the solution in this form is that it involves complex-valued functions. It is possible to re-express the general solution in terms of two linearly independent real-valued functions. Using this identity, we have: This yields the solution: Both of these functions are solutions to the original ode. In addition, they are linearly independent, since they are not multiples of each. Hence, any solution to the ode can be expressed in terms of these function. So we can write: Hence, the general solution can be written:

Chapter 6 : Linear differential equation - Wikipedia

Also note that a secondâ€•order linear homogeneous differential equation with constant coefficients will always give rise to a secondâ€•degree auxiliary polynomial equation, that is, to a quadratic polynomial equation.

Chapter 7 : Linear Non-homogeneous Ordinary Differential Equations with Constant Coefficients

equation with constant coefficients (that is, when p(t) and q(t) are constants). Since a homogeneous equation is easier to solve compares to its nonhomogeneous counterpart, we start with second order linear.

Chapter 8 : Ordinary differential equation - Wikipedia

In the case of nonhomgeneous equations with constant coefficients, the complementary solution can be easily found from the roots of the characteristic polynomial.

Chapter 9 : Differential Equations - Basic Concepts

In mathematics, an ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and its calendrierdelascience.com term ordinary is used in contrast with the term partial differential equation which may be with respect to more than one independent variable.