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Chapter 1 : Daniel Quillen - Wikipedia

In mathematics and specifically in topology, rational homotopy theory is a simplified version of homotopy theory for topological spaces, in which all torsion in the homotopy groups is ignored.

Adams, On the cobar construction,Proc. Gugenheim, On PL deRham theory and rational homotopy type, preprint. Sullivan, Real homotopy theory of Kdhler manifolds,Invent. Moore, Foundations of relative homological algebra, Amer. Moore, Homology and fibrations I. Moore,Limits and spectral sequences, Topology, 1 , Morgan, Homotopy theory anddifferential forms, Seminorio di Geometria, Firenze, mimeographed. Stasheff, Differentialhomological algebra and homogeneous spaces, J. Algebra, 5 , Lazard, Sur les groupes nilpotentset les anneaux de Lie, Ann. Malcev, Nilpotent groups without torsion, Izvest. Miller, Minimal Lie algebras in rational homotopy theory, Universityof Notre Dame dissertation, Neisendorfer, Formal and coformal spaces, preprint. Moore, On the structure of Hopf algebras, Ann. Moore, Differentialhomological algebra, Actes, Congres, Intern. Berlin, Heidelburg, New York: Moore,What it means for a coalgebra to be simply connected, preprint. Neisendorfer, Rational homotopy groups of complete intersections, preprint. Dolbeault homotopy theory, preprint. Quillen, Rational homotopy theory, Ann. Smith, Homological algebra and the Eilenberg-Moore spectral sequence, Trans. Soc, ,

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Chapter 2 : Rational homotopy theory and differential forms (eBook,) [calendrierdelascience.com]

RATIONAL HOMOTOPY THEORY 3 It is clear that for all r, Sn r is a strong deformation retract of X(r), which implies that HkX(R) = 0 if k 6= 0,calendrierdelascience.comrmore, the homomorphism induced in reduced.

This may seem like a rather silly point to make, but the advantage is that the latter class of maps is easier to characterize algebraically. Furthermore, two such maps are homotopic if and only if they induce the same isomorphism. Hence, to form the homotopy category, we can identify maps which induce the same isomorphism on homotopy groups. This is usually done by adding arrows to the category which are formal inverses to such classes of maps. This leads naturally to the definition of rational homotopy. Given a space , we can throw away all the torsion information in by tensoring with ; ie, looking at the rational homotopy group. Thus, we can form the rational homotopy category by adding formal inverses to rational homotopy equivalences. This is a coarser equivalence relation on spaces than you might think. To get a sense for what is going on here, think about any finite-sheeted covering map. This induces an isomorphism on all , and a surjection on with finite order kernel. Therefore, it is an isomorphism on all rational homotopy groups and hence a rational homotopy equivalence. So rational homotopy is blind to quotienting by the free action of a finite group. Take, for instance, the study of hyperbolic Riemann surfaces. By the Riemann mapping theorem , these are all quotients of the hyperbolic disc by some subgroup of a Fuchsian group. A useful relationship between two Fuchsian groups is that of commensurablity , that is, having intersection which is cofinite in each group. By the same argument as the previous paragraph, two Riemann surfaces with commensurable Fuchsian groups are rational homotopy equivalent and the converse is also true. A map induces an isomorphism on all rational homotopy groups ie, is a rational homotopy equivalence if and only if the map induced on all rational cohomology groups is an isomorphism. Therefore, we can go back and redefine the rational homotopy category by inverting morphisms which induce an isomorphism on all rational cohomology groups. To any space we can assign its cochain complex. This determines a functor from the category of spaces to the category of differential graded algebras; specifically DGAs which are commutative and have no negative-degree terms. Then the rational homotopy category of spaces maps into the category of topological DGAs modulo quasi-isomorphism. What is even better is that if we restrict to simply connected, finite dimensional spaces , and topological DGAs which are finite dimensional in each degree and have zero first cohomology, then this functor is an equivalence of categories. This allows us to abandon spaces entirely and work with DGAs. I find this personally very satisfying, since I already know several circumstances where it is useful to think of DGAs as almost like spaces, and this theorem lets me know exactly what is lost when thinking like this its the information inaccessible to rational homotopy theory. This also transitions into one of the most fun aspects of math at least for me personally , which is taking an equivalence and seeing what natural constructions on one side look like on the other side. If the mood strikes me, I might write a follow-up post to these outlining some of these constructions. Related This entry was posted on April 27, at 5: You can follow any responses to this entry through the RSS 2. You can leave a response , or trackback from your own site.

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Chapter 3 : Newest 'rational-homotopy-theory' Questions - MathOverflow

The natural setting of algebraic topology is the homotopy category. Restricting attention to simply-connected homotopy types and mappings between them allows the algebraic operation of localization (cf. Localization in categories). Inverting all the primes yields rational homotopy theory. One simple.

There are examples of non-isomorphic minimal Sullivan models with isomorphic cohomology algebras. The Sullivan minimal model of a topological space[edit] For any topological space X, Sullivan defined a commutative differential graded algebra APL X , called the algebra of polynomial differential forms on X with rational coefficients. An element of this algebra consists of roughly a polynomial form on each singular simplex of X, compatible with face and degeneracy maps. This algebra is usually very large uncountable dimension but can be replaced by a much smaller algebra. More precisely, any differential graded algebra with the same Sullivan minimal model as APL X is called a model for the space X. When X is simply connected, such a model determines the rational homotopy type of X. This is called the Sullivan minimal model of X; it is unique up to isomorphism. The rational cohomology of the space is the cohomology of its Sullivan minimal model. The spaces of indecomposables in V are the duals of the rational homotopy groups of the space X. The Whitehead product on rational homotopy is the dual of the "quadratic part" of the differential d. Two spaces have the same rational homotopy type if and only if their minimal Sullivan algebras are isomorphic. When X is a smooth manifold, the differential algebra of smooth differential forms on X the de Rham complex is almost a model for X; more precisely it is the tensor product of a model for X with the reals and therefore determines the real homotopy type. One can go further and define the p-completed homotopy type of X for a prime number p. This is equivalent to requiring that the cohomology algebra of A viewed as a differential algebra with trivial differential is a model for A though it does not have to be the minimal model. Thus the rational homotopy type of a formal space is completely determined by its cohomology ring. For manifolds, formality is preserved by connected sums. On the other hand, closed nilmanifolds are almost never formal: Closed symplectic manifolds need not be formal: There are also examples of non-formal, simply connected symplectic closed manifolds. Indeed, if a differential graded algebra A is formal, then all higher order Massey products must vanish. The converse is not true: The complement of the Borromean rings is a non-formal space: It has a basis of elements 1, u,u2, Then this algebra is a minimal Sullivan algebra that is not formal. So V cannot be a model for its cohomology algebra. The corresponding topological spaces are two spaces with isomorphic rational cohomology rings but different rational homotopy types.

Chapter 4 : CiteSeerX â€” Citation Query Rational homotopy theory

The rational LS category is the LS category of a rational CW complex in the rational homotopy type of the space, and the authors calculate it in terms of Sullivan models, verifying that the rational case is much easier to compute than the general case.

Chapter 5 : Neisendorfer : Lie algebras, coalgebras and rational homotopy theory for nilpotent spaces.

Rational Homotopy Theory I Dylan Wilson February 4, (1) As homotopy theorists we tend to understand spaces by breaking them up into easy to understand pieces.

Chapter 6 : [math/] Rational homotopy theory: a brief introduction

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Chapter 7 : rational homotopy theory in nLab

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Rational homotopy theory is the study of spaces up to rational homotopy equivalence. There are two seminal papers in the subject, Quillen's [20] and Sullivan's [25].

Chapter 8 : Sullivan model of free loop space in nLab

Interactions between homotopy theory and algebra / Summer School on Interactions between Homotopy Theory and Algebra, University of Chicago, July August 6, , Chicago, Illinois,

Chapter 9 : Infinity structures and higher products in rational homotopy theory

Idea. Rational homotopy theory is the homotopy theory of rational topological spaces, hence of rational homotopy types: simply connected topological spaces whose homotopy groups are vector spaces over the rational numbers.