DOWNLOAD PDF RESOLUTION OF SINGULARITIES EDWARD BIERSTONE AND PIERRE D. MILMAN

Chapter 1 : CiteSeerX â€” Resolution of singularities

Bierstone, Edward; Milman, Pierre D. (), "Functoriality in resolution of singularities", Publications of the Research Institute for Mathematical Sciences, 44 (2).

In practice it is more convenient to ask for a different condition as follows: More generally, it is often useful to resolve the singularities of a variety X embedded into a larger variety W. Suppose we have a closed embedding of X into a regular variety W. The map from the strict transform of X to X is an isomorphism away from the singular points of X. It cannot be made functorial for all not necessarily smooth morphisms in any reasonable way. Or in general, the sequence of blowings up is functorial with respect to smooth morphisms. Hironaka showed that there is a strong desingularization satisfying the first three conditions above whenever X is defined over a field of characteristic 0, and his construction was improved by several authors see below so that it satisfies all conditions above. Resolution of singularities of curves[edit] Every algebraic curve has a unique nonsingular projective model, which means that all resolution methods are essentially the same because they all construct this model. In higher dimensions this is no longer true: This can be done over more general fields by using the set of discrete valuation rings of the field as a substitute for the Riemann surface. This method extends to higher-dimensional varieties, and shows that any n-dimensional variety has a projective model with singularities of multiplicity at most n! Normalization removes all singularities in codimension 1, so it works for curves but not in higher dimensions. Valuation rings[edit] Another one-step method of resolving singularities of a curve is to take a space of valuation rings of the function field of the curve. This space can be made into a nonsingular projective curve birational to the original curve. Blowing up Repeatedly blowing up the singular points of a curve will eventually resolve the singularities. The main task with this method is to find a way to measure the complexity of a singularity and to show that blowing up improves this measure. There are many ways to do this. For example, one can use the arithmetic genus of the curve. Eventually this produces a plane curve whose only singularities are ordinary multiple points all tangent lines have multiplicity 1. It starts with a plane curve, and repeatedly applies birational transformations to the plane to improve the curve. Resolution of singularities of surfaces[edit] Surfaces have many different nonsingular projective models unlike the case of curves where the nonsingular projective model is unique. However a surface still has a unique minimal resolution, that all others factor through all others are resolutions of it. In higher dimensions there need not be a minimal resolution. There were several attempts to prove resolution for surfaces over the complex numbers by Del Pezzo , Levi , Severi , Chisini , and Albanese , but Zariski , chapter I section 6 points out that none of these early attempts are complete, and all are vague or even wrong at some critical point of the argument. The first rigorous proof was given by Walker , and an algebraic proof for all fields of characteristic 0 was given by Zariski Abhyankar gave a proof for surfaces of non-zero characteristic. Resolution of singularities has also been shown for all excellent 2-dimensional schemes including all arithmetic surfaces by Lipman Although this will resolve the singularities of surfaces by itself, Zariski used a more roundabout method: For surfaces this reduces to the case of singularities of order 2, which are easy enough to do explicitly. The hardest case is valuation rings of rank 1 whose valuation group is a nondiscrete subgroup of the rational numbers. In particular if Y is excellent then it has a desingularization. His method was to consider normal surfaces Z with a birational proper map to Y and show that there is a minimal one with minimal possible arithmetic genus. He then shows that all singularities of this minimal Z are pseudo rational, and shows that pseudo rational singularities can be resolved by repeatedly blowing up points. Resolution of singularities in higher dimensions[edit] The problem of resolution of singularities in higher dimensions is notorious for many incorrect published proofs and announcements of proofs that never appeared. He first proved a theorem about local uniformization of valuation rings, valid for varieties of any dimension over any field of characteristic 0. He then showed that the Zariskiâ€"Riemann space of valuations is quasi-compact for any variety of any dimension over any field , implying that there is a finite family of models of any projective variety such that any valuation has a smooth center over at least one of these models. The final and hardest part of the proof, which uses the fact that the variety is of dimension 3 but which works for all characteristics, is to show that given 2 models one can find a third that resolves the singularities that each of the two given models resolve. He proved that it was possible to resolve singularities of varieties over fields of characteristic 0 by repeatedly blowing up along non-singular subvarieties, using a very complicated argument by induction on the dimension. For an expository account of the theorem, see Hauser and for a historical discussion see Hauser De Jong proved that for any variety X over a field there is a dominant proper morphism which preserves the dimension from a regular variety onto X. This need not be a birational map, so is not a resolution of singularities, as it may be generically finite to one and so involves a finite extension of the function field of X. Resolution for schemes and status of the problem[edit] It is easy to extend the definition of resolution to all schemes. Not all schemes have resolutions of their singularities: Grothendieck , section 7. Grothendieck also suggested that the converse might hold: Hauser gave a survey of work on the unsolved characteristic p resolution problem. Method of proof in characteristic zero[edit] The lingering perception that the proof of resolution is very hard gradually diverged from reality. In every case the global object the variety to be desingularized is replaced by local data the ideal sheaf of the variety and those of the exceptional divisors and some orders that represents how much should be resolved the ideal in that step. With this local data the centers of blowing-up are defined. The centers will be defined locally and therefore it is a problem to guarantee that they will match up into a global center. This can be done by defining what blowings-up are allowed to resolve each ideal. Done appropriately, this will make the centers match automatically. Another way is to define a local invariant depending on the variety and the history of the resolution the previous local centers so that the centers consist of the maximum locus of the invariant. The definition of this is made such that making this choice is meaningful, giving smooth centers transversal to the exceptional divisors. In either case the problem is reduced to resolve singularities of the tuple formed by the ideal sheaf and the extra data the exceptional divisors and the order, d, to which the resolution should go for that ideal. This tuple is called a marked ideal and the set of points in which the order of the ideal is larger than d is called its co-support. The proof that there is a resolution for the marked ideals is done by induction on dimension. The induction breaks in two steps: A key ingredient in the strong resolution is the use of the Hilbertâ€”Samuel function of the local rings of the points in the variety. This is one of the components of the resolution invariant. Examples[edit] Multiplicity need not decrease under blowup[edit] The most obvious invariant of a singularity is its multiplicity. However this need not decrease under blowup, so it is necessary to use more subtle invariants to measure the improvement. In the previous example it was fairly clear that the singularity improved since the degree of one of the monomials defining it got smaller. This does not happen in general. It is not immediately obvious that this new singularity is better, as both singularities have multiplicity 2 and are given by the sum of monomials of degrees 2, 3, and 4. Blowing up the most singular points does not work[edit] Whitney umbrella A natural idea for improving singularities is to blow up the locus of the "worst" singular points. However blowing up the origin reproduces the same singularity on one of the coordinate charts. So blowing up the apparently "worst" singular points does not improve the singularity. Instead the singularity can be resolved by blowing up along the z-axis. After the resolution the total transform, the union of the strict transform, X, and the exceptional divisors, is a variety with singularities of the simple normal crossings type. Then it is natural to consider the possibility of resolving singularities without resolving this type of singularities, this is finding a resolution that is an isomorphism over the set of smooth and simple normal crossing points. When X is a divisor, i. Incremental resolution procedures need memory[edit] A natural way to resolve singularities is to repeatedly blow up some canonically chosen smooth subvariety. This runs into the following problem. The only reasonable varieties to blow up are the origin, one of these two axes, or the whole singular set both axes. However the whole singular set cannot be used since it is not smooth, and choosing one of the two axes breaks the symmetry between them so is not canonical. This means we have to start by blowing up the origin, but this reproduces the original singularity, so we seem to be going round in circles. The solution to this problem is that although blowing up the origin does not change the type of the singularity, it does give a subtle improvement: However, in order to exploit this the resolution procedure needs to treat these 2 singularities differently, even though they are locally the same. This is sometimes done by giving the resolution procedure some memory, so the center of the blowup at each step depends not only on the singularity, but on the previous blowups used to produce it. However it is not possible to find a strong resolution functorial for all possibly non-smooth morphisms. The XY-plane is already nonsingular so should not be changed by resolution, and any resolution of the conical singularity factorizes through the minimal resolution given by blowing up the singular point. However the rational map from the XY-plane to this blowup does not extend to a regular map. Minimal resolutions need not exist[edit] Minimal resolutions resolutions such that every resolution factors through them exist in dimensions 1 and 2, but not always in higher dimensions. The Atiyah flop gives an example in 3 dimensions of a singularity with no minimal resolution. Singularities of toric varieties[edit] Singularities of toric varieties give examples of high-dimensional singularities that are easy to resolve explicitly. A toric variety is defined by a fan, a collection of cones in a lattice. The singularities can be resolved by subdividing each cone into a union of cones each of which is generated by a basis for the lattice, and taking the corresponding toric variety. Choosing centers that are regular subvarieties of X[edit] Construction of a desingularization of a variety X may not produce centers of blowings up that are smooth subvarieties of X. Many constructions of a desingularization of an abstract variety X proceed by locally embedding X in a smooth variety W, considering its ideal in W and computing a canonical desingularization of this ideal. The desingularization of ideals uses the order of the ideal as a measure of how singular is the ideal. The desingularization of the ideal can be made such that one can justify that the local centers patch together to give global centers. However, this method only ensures centers of blowings up that are regular in W. Therefore, the resulting desingularization, when restricted to the abstract variety X, is not obtained by blowing up regular subvarieties of X. Other variants of resolutions of singularities[edit] After the resolution the total transform, the union of the strict transform, X, and the exceptional divisor, is a variety that can be made, at best, to have simple normal crossing singularities. Then it is natural to consider the possibility of resolving singularities without resolving this type of singularities. The problem is to find a resolution that is an isomorphism over the set of smooth and simple normal crossing points. The general case or generalizations to avoid different types of singularities are still not known. Avoiding certain singularities is impossible.

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