## Chapter 1 : Exponential Stability of Nonautonomous Infinite Dimensional Systems

*Stability of nonautonomous equations in Hilbert spaces Setup Here we consider nonlinear perturbations v prime =A(t)v+f(t,v)of the linear equation v prime = A(t)v, and study the persistence of the stability of solutions under sufï¬•ciently small perturbations.*

In the theory of control systems, controllability and stability are the qualitative control problems that play an important role in the systems. The theory was first introduced by Kalman et al. On its development, the theory can be generalized into controllability and stabilizability of the nonautonomous time-varying control systems see, e. Idea of this problem is to find an admissible control such that the corresponding solution of the system has desired properties. There are many various qualitative control problems that can be implemented to study the stabilizability. One of the most commonly applied qualitative control problems is null controllability. The system 1 is said to be null controllable if there exists an admissible control which steers an arbitrary state of the system into. The associated stabilizability problem is to find a control such that the zero solution of the closed-loop system is asymptotically stable in the Lyapunov sense. In this context, the system is said stabilizable and is called the stabilizing feedback control. The complete stabilizability is one of various types of stability that is often applied to characterize the stability of the control systems. This term was first introduced by Wonham [ 5 ] which relates to exponential stability of the systems. Next, based on the Lyapunov function techniques, Phat [ 3 ] investigated that the null controllability guaranteed the output feedback stabilization for the non-autonomous systems. While Jerbi [ 6 ] deal the problem of stabilizability at the origin of a homogeneous vector field of degree three. However, it does not hold for the converse. Furthermore, if the system is completely stabilizable, then it is null controllable in finite time. Investigations of controllability and stabilizability in the infinite dimensional control theory are more complicated, in particular for nonautonomous systems. For non-autonomous control systems of the finite-dimensional spaces, Ikeda et al. As extension of the some results of [ 7 ], Phat and Ha [ 4 ] characterized the controllability via the stabilizability and Riccati equation for the linear nonautonomous systems. The results of the stabilizability for the finite-dimensional systems can be generalized into infinite-dimensional systems. For the autonomous systems, Phat and Kiet [ 8 ] investigated relationship between stability and exact null controllability extending the Lyapunov equation in Banach spaces. The smart characterization of generator of the perturbation semigroup for Pritchard-Salamon systems was provided by Guo et al. The unbounded feedback is also investigated in the paper. For nonautonomous systems, Hinrichsen and Pritchard [ 11 ] introduced a concept of radius stability for the systems under structured nonautonomous perturbations. Indeed, this concept is an advanced investigation of the stabilizable theory. In the linear nonautonomous systems in Hilbert spaces, Niamsup and Phat [ 12 ] have proved that exact null controllability implies the complete stabilizability. Fu and zhang [ 13 ] had established a sufficient result of exact null controllability for a nonautonomous functional evolution system with nonlocal conditions using theory of linear evolution system and Schauder fixed point theorem. As described in our recent work [ 14 ], a quasisemigroup is an alternative approach that can be implemented to investigate the non-autonomous systems 1. This approach was first introduced by Leiva and Barcenas [ 15 ]. By this approach, is an infinitesimal generator of a -quasisemigroup on. Even, the quasisemigroup approach can be applied to characterize the controllability of the non-autonomous control systems, although it is still limited to the autonomous controls [ 18 ]. However, until now there is no research which investigates the qualitative control problems of the nonautonomous control systems implementing -quasisemigroup theory. In this paper, we are concern on the exact null controllability, stability, stabilizability, complete stabilizability, detectability, and possible relationship among them. In paper, we identify the stability with the uniform exponential stability of the associated -quasisemigroup. The organization of this paper is as follows. In Section 2 , we provide the sufficient and necessary conditons for uniform exponential stability of -quasisemigroup which is an extension of [ 17 ]. Relationships among stability, stabilizability, and detectability of the linear nonautonomous control systems are considered in Section 3. In Section 4 , we discuss connection between exact null controllability and complete stabizability of the linear nonautonomous control systems. Uniform Exponentially Stability of -Quasisemigroups This section is a part of the main results. We first recall the definition of a strongly continuous quasisemigroups following [ 15 , 18 ]. Let be the set of all bounded linear operators on a Hilbert space. A two-parameter commutative family is called a strongly continuous quasisemigroup, in short -quasisemigroup, on.

## Chapter 2 : On Almost Automorphic Mild Solutions for Nonautonomous Stochastic Evolution Equations

*We introduce a large class of nonautonomous linear differential equations v =A(t)vin Hilbert spaces, for which the asymptotic stability of the zero solution, with all Lyapunov exponents of the linear equation negative, persists in v = A(t)v + f(t,v) under sufï¬•ciently small perturbations.*

## Chapter 3 : A Perron-type theorem for nonautonomous difference equations - IOPscience

*12 Stability of nonautonomous equations in Hilbert spaces We study in this chapter the persistence of the asymptotic stability of the zero solution of a nonautonomous linear equation v = A(t)v under a pertur-.*

## Chapter 4 : Breda : Nonautonomous delay differential equations in Hilbert spaces and Lyapunov exponents

*Main theme of this volume is the stability of nonautonomous differential equations, with emphasis on the Lyapunov stability of solutions, the existence and smoothness of invariant manifolds, the.*